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RESEARCH ARTICLE

Fama and French (2015) five-factor model using SEM with a Mediating Role of Liquidity: Evidence from Pakistan

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Abstract

Liquidity is one of the intricate phenomena that cannot be assessed in a single dimension due to its multidimensional structure, which is still contentious among researchers and must be explored from several perspectives. This study thus analyses the multidimensional liquidity as mediating variable to empirically investigate whether liquidity influence the nexus between risk-premiums and portfolio stock returns using Structural Equation Modeling. Using liquidity as factor is employed using time-series OLS regression technique. The sample used in this study comprised of monthly returns of 286 non-financial firms enlisted on PSX for time span from January 2006 through June 2022. The findings of the study reveal that liquidity as mediating variable performs statistically highly significant while as independent risk-factor also performs statistically highly significant using Fama and French (2015) five-factor model. The market risk-premium exhibits statistically insignificant results for PSX while size, profitability and investment also show significant findings in the market. The potential investors and portfolio managers need to consider liquidity as benchmark criteria prior to make decision regarding investing in PSX.

Keywords: Liu (2006) multidimensional liquidity; Fama and French (2015) five-factor model; Structural Equation Modeling; OLS regression; Pakistan Stock Exchange

Introduction

Stock markets play a dynamic role and directly contribute to higher economic growth of a country (Afonso, & Reimers, 2022) where stocks are traded on a regular basis by mobilising domestic resources (Husain & Mahmood, 2001) and shifting money from savers to investors (Marques, Fuinhas & Marques, 2013; Oskooe, 2010; Deb & Mukherjee, 2008). The individual investors and portfolio managers are extremely sensitive of their rational investment decision-making about valuation and allocation of financial securities in building their internationally diversified portfolios in such a dynamic, volatile, and illiquid stock market. However, idiosyncratic risk, which is linked to uncertain future expected returns, influences their investment decisions (Ullah, et al., 2019). The stock investors are rationally hypercautious about prospective returns and stock liquidity from a psychosomal perspective, which drives them to be rational before making investments in the stock markets.

Moreover, previous literature documents an inverse nexus between liquidity and average portfolio/stock returns in stock markets such as Hartian, and Sitorus, (2015) observed negative nexus between liquidity and stock returns in developed countries using trading volume, turnover ratio, and turnover volatility as proxy for liquidity. Due to multidimensional nature of liquidity, several proxies are employed to quantify the liquidity and examine its nexus with portfolio/stock returns; nevertheless, it is still vague which proxy is best suited for which emerging equity market. There is a plethora of studies employed numerous proxies of liquidity to demonstrate direct nexus with equity returns, but to the best of my knowledge, no study has used it as a mediating variable using asset pricing models (APMs). Irom, Ibiamke, and Nyor (2022) empirically examined the relationship between capital structure and stock returns while augmenting liquidity as a mediating variable in their study; thus, the study differs by two perspectives, including the use of Liu (2006) multidimensional liquidity as a mediating variable in addition to the study of Fama and French (2015) five-factor model as suggested by Azam (2022a). Due to the ability of measuring the multifaceted dimension of liquidity, Liu (2006) model can estimate better coefficients, hence this study choose as mediating variable between portfolio returns and risk-premiums to analyze the PSX market.

Previous research has examined the validity of APMs on explaining portfolio/stock returns on the Pakistan Stock Exchange and also examined liquidity augmented various APMs and also used GDP-Growth as mediating variable using CAPM but has not been used multidimensional liquidity as mediating variable using Fama and French (2015) five-factor model specifically for Pakistan Stock Exchange. A sample of 286 enlisted firms was used in this study to construct 25 value-weighted 25 portfolios based Size-B|M ratios in accordance with Fama and French (1993; 2015) in order to determine the shortcomings and gaps of the FF5FM and Azam (2021) six-factor model (A6FM) in this market. We examined FF5FM using Structural Equation Modeling (SEM) using Liu (2006) multidirectional liquidity as a mediating variable to investigate whether liquidity mediates the impact of five-factors on portfolio returns of non-financial firms' portfolios. In most previous researches, time-series multivariate regression analyses are employed while this study uses both SEM with mediation of liquidity and OLS regression technique with liquidity as augmented factor to evaluate the efficiency and impact of each factor on the portfolio returns to assess direct and indirect influence of each factor. These situations, however, were raised in the current investigation for PSX.

Moreover, liquidity is one of the intricate phenomena that cannot be measured in a single dimension due to its multidimensional structure, which is still contentious among researchers and must be explored from several perspectives. This study thus analyses the multidimensional liquidity as mediating variable to empirically investigate whether liquidity influence the nexus between risk-premiums and portfolio stock returns using Structural Equation Modeling. The following research questions are addressed in this study: Does liquidity explain portfolio returns as a mediating or independent variable? Do market, size, value, profitability, and investment factor account for PSX portfolio returns? In explaining portfolio returns in PSX, does Azam (2021) six-factor model outperform Fama and French (2015) five-factor model?

To better understand the effects of liquidity on stock returns and the numerous factors affecting portfolio returns in PSX, the study aims to provide answers to the following questions. First, considering several measurements, is there a substantial relationship between liquidity and stock returns? The direct relationship between multidimensional liquidity and stock returns are empirically tested in PSX such as Azam (2021) but are there any mediating effect of liquidity on stock returns using Fama and French (2015) five-factor model? Second, to what extent do portfolio stock returns depend on the market, size, value, profitability, and investment factors?

Literature Review

The empirically tested discipline of APMs spanning developed, emerging, and frontier markets throughout the world is one of the most actively researched fields in the field of investing and portfolio management that is yet inconclusive and progressing among scholars. Among various APMs, the most thoroughly investigated model is Fama and French (1993) three-factor model, which is augmented with two additional factors such as profitability and investment and is known as Fama and French (2015) five-factor model (henceforth FF5FM) and assumed to be the dominant models of asset pricing (Singh, Singh, & Prakash, 2022). It is also scrutinized around the globe using various stock markets. The findings of FF-5FM showed diversified results in terms of validity and performance in explaining potential average portfolio stock returns.

According to recent studies, relying solely on a single factor model (CAPM) exclusively might lead to substantial losses. As a consequence, studies like those by Zaremba, et al. (2019) looked at 160 anomalies in order to thoroughly analyse which factor substantially influences the average expected returns of stocks using dataset of 23 Frontier Markets. More recently, Azam (2022c) has investigated Tobin-q as a risk-premium augmented with CAPM, FF3FM, C-4FM, and FF5FM utilising the PSX dataset for time-span from 1994 to 2020. The study calculated excess portfolio and market returns using the 3-month T-bill rate as the risk-free rate. They employed monthly data from 521 financial and nonfinancial firms to perform a comprehensive analysis on PSX utilising the time-series OLS regression approach. The findings demonstrate statistically significant determinants such as size, value, profitability, and Tobin-q risk-factor, whereas market and investment show negligible findings. The GRS test found the Tobin-q augmented FF-5FM to be the market's most efficient model. Literature also shows that liquidity, as a mediating variable, may not provide superior results such as Irom, Ibiamke, and Nyor (2022) investigated the capital structure and stock returns nexus using Nigerian stock market data in order to gain insight into the mediating function of liquidity proxy and provide a fresh viewpoint to the current research. The liquidity ratio as the proxy of liquidity has been assumed but it is observed that liquidity has no significant impact on stock returns. The study further revealed that liquidity has moderate nexus with capital structure and stock returns.

Recent asset pricing literature provides new empirical evidence that stock returns are valued for related risk characteristics such as market beta, size, value, profitability, and investment, whereas Azam (2021) enhanced liquidity as the most important risk factor in PSX. The size, investment, and liquidity anomalies in the Azam (2021) liquidity augmented six-factor model are discovered to be the highly statistically significant anomalies using OLS regression technique, similarly the value and investment factors in the Fama and French (2015) five-factor model. Haddad and Hellara (2019) used Hu, Pan and Wang (2013) liquidity proxy and observed significant and negative nexus with stock returns in emerging markets.

Liquidity is a major aspect of the financial market and is critical for investment strategies and financial securities (Irom, Ibiamke, & Nyor, 2022). Ma, Anderson, and Marshall (2016) described that liquidity has a dynamic impact on the financial market, impacting equity returns and fostering stability and growth in the capital markets (Nguyen & Puri, 2009). It is the primary variable determining the efficiency of the capital markets (Ye, et al., 2021). The liquidity of a financial asset, on the other hand, is one of the desired qualities that investors want (Minovi, Stevanovi, & Belopavlovi, 2011). It is a vital component of today's financial markets (Scharnowski, 2021). A lack of liquidity can have a detrimental influence on stock prices (Irom, Ibiamke, & Nyor, 2022). Additionally, they look for asymmetric information that may have an impact on stock prices directly or indirectly in order to protect their initial investments and expected returns. The Efficient Market Hypothesis (EMH) theory, in contrast, asserts that stock prices accurately represent all information that is accessible in efficient markets. Since the last 50 years, a plethora of variables, titled the "zoo of factors" (Cochrane, 2011; 2017), have been proposed and empirically investigated throughout the world to challenge the EMH paradigm and to support the idea that there are many determinants that investors and portfolio managers can use to diversify idiosyncratic risk and outperform the market. Besides, anticipating estimated future returns is not transcendental, which cannot be found in the contemporary period when economists and statisticians created computer-assisted instruments and methodologies. Many academics and researchers have worked hard to build ideas and useful models to forecast predicted stock returns in this area. These advancements have as their justification the best assessment and selection of financial assets or optimal portfolio with a view to maximise the associated expected returns and reduce the idiosyncratic risk correlating with their prudent investment (Garvey, Murphy, & Wu, 2007).

Furthermore, Amihud and Mendelson (1986a) included the liquidity factor, a vital indicator of financial security, as a new variant in the zoo of factors. One cluster, however, treated liquidity as a factor loading and treated it as illiquid minus liquid (IML) firms (Liu 2006), whereas another estimated it as a reliable indicator of stock returns in their studies and discovered statistically significant (Acharya & Pederson, 2005; Paster & Stambaugh, 2003; Liu, 2006) and robust contributions utilising multivariate analysis (Belkhir, Saad, & Samet, 2020; Grillini et al., 2019; Wu, 2019; Abdi & Ranaldo, 2017). Similar findings were observed by (Chan & Faff, 2005; Javid & Ahmed, 2008) who found a statistically significant and established linkage between the illiquidity premium and predicted returns. As a result, this study is the first to examine liquidity's unexpected impact on stock returns using liquidity as a mediating variable with Fama and French's (2015) five-factor model in PSX due to the multidimensional nature of liquidity, which makes it impossible to assess in a one-dimensional manner.

The literature suggests that multifactor APMs were expected to outperform single-factor APMs in explaining the variation in portfolio returns (Chen, 1983; Ross, 1976; Brown & Weinstein, 1983; Burmeister & Wall, 1986). As claimed by Fama and French (2015), the FF-5FM outperforms previous single-factor and multiple factor APMs in explaining between 71-94 percent of volatility in equity returns in the US, although it still lacks complete capturing ability (Lin, 2017). Despite decades of study, it is still ambiguous what causes cross-sectional price fluctuations in anticipated stock returns (Annaert, De Ceuster, & Verstegen, 2013). In a similar vein, Huang (2019) doubts the model's reliability and explanatory ability in emerging equities markets. Therefore, more investigation of increased FF-5FM in Pakistan's expanding market is necessary to determine the authenticity of the findings. Azam (2022d) empirically examined the FF5FM with additional determinant of Consumer Confidence

Index (CCI) using non-financial sectors of PSX. The findings revealed statistically insignificant outcomes for CCI while FF5FM produces significant explanatory power in the market.

There is a plethora of studies conducted on the valuation of APMs during Covid-19 and its influence on stock volatility using ARCH and GARCH such as Azam, & Azeem (2021) using PSX dataset and observed significant impact. On the other hand, Azam and Ilyas (2011) examined Price to earnings (P/E) premium and leverage premium and observed significant estimates using PSX dataset. Similarly, Qadeer et al. (2022) used PSX data and analyzed FF-5FM in augmenting turnover as liquidity proxy and observed significant results but it is single dimensional. Younus and Butt (2022) examined 54 anomalies using sample of 290 firms' dataset. The time-series tests are conducted using various APMs on PSX but vague results were observed to explain the portfolio returns. On the other hand, using FF-5FM in PSX, Zada, Rehman, and Khwaja (2018) observed size, value, profitability and investment are priced by the market using Fama and MacBeth (1973) two-steps regression technique by assuming a sample of 120 firms. On the other hand, Thafani & Ediriwickrama (2022) empirically examined the FF-5FM in Sri Lankan equity market using time span from June 2009 to December 2018. The model employed is Newey and West (1987) weighted average least square regression model Sri Lankan stock exchange. The size, profitability and investment are redundant in the market. The results showed similarity with Fama and French (2017) for Japan and Asia Pacific portfolios.

After going through the literature review, the study extracts the following hypotheses to be investigated:

H_1: Liquidity as mediator has indirectly significant nexus between the risk-factors and portfolio returns.

H_2: Liquidity as risk-factor has directly significant nexus with the portfolio returns.

Data and Methodology

To analyse the Pakistan stock market, estimates are generated using the FF5FM through Structural Equation Modeling (SEM) with multidimensional liquidity as a mediating variable, as well as OLS multivariate regression with liquidity as the independent variable. The monthly returns of the PSX-100 Index are used as market returns, and the 3-month Treasury bills rate is utilised as the risk-free rate, to compute the excess market returns. The Fama and French (2015) sampling criteria were employed, and all non-financial 286 firms were included; however, firms with negative B|M were excluded.

The primary goal of this research is to empirically determine the validity of the Fama and French (2015) five-factor model and multidimensional liquidity augmented Azam (2021) six-factor model in explaining the volatility of portfolio returns in the Pakistan equity market. To carry out the test, the following empirical models are used: Structural Equation Modeling (SEM) and OLS multivariate regression. Multidimensional liquidity is utilised as a mediating variable in SEM, whereas it is employed as an independent variable in OLS regression to investigate which factor has explanatory power for portfolio returns in PSX by following Azam (2022d) who empirically used the Consumer Confidence Index in the same vein. SEM is a method for a diverse set of approaches used by researchers in both empirical and observational study throughout the disciplines Boslaugh et al. (2008).

Model Specification

The SEM and multivariate time-series OLS regression are the novel frameworks used in this study, based on the follows specifications:

Fama & French (2015) five-factor model (FF5FM)

$$R_i - R_f = R_f + \beta_m(R_m - R_f) + \beta_s(SmB) + \beta_v(HmL) + \beta_p(RmW) + \beta_i(CmA) + \varepsilon_i \qquad (1)$$

Where, R_i - R_f, is excess returns of portfolio. R_m - R_f, is the excess returns of market. SmB is the Small minus Big firms returns called Size factor. HmL is the High minus Low firms returns called Value factor. RMW is the Robust minus Weak firms returns called Profitability factor. CMA is the Conservative minus Aggressive firms returns called Investment factor. β_m, β_s, β_v, β_p, and β_i are the coefficients of market, size, value, profitability and investment factors respectively.

Azam (2021) six-factor model (A-6FM)

$$R_i - R_f = R_f + \beta_m(R_m - R_f) + \beta_s(SmB) + \beta_v(HmL) + \beta_p(RmW) + \beta_i(CmA) + \beta_l(ImL) + \varepsilon_i \qquad (2)$$

Where, IML is the Illiquidity minus Liquidity firms' returns called (Liu, 2006) multidimensional liquidity factor. β_m, β_s, β_v, β_p, β_i and β_l are the coefficients of market, size, value, profitability, investment and liquidity factors respectively.

Portfolio Construction

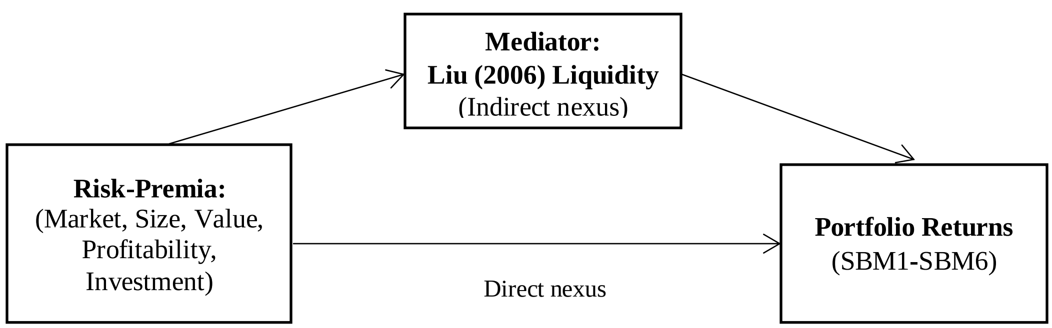
Diversification, according to portfolio theory, boosts returns by including uncorrelated assets in the portfolio; hence, portfolio returns outperform investment in individual equities (Jones & Trevillion, 2022). This study used size-B|M ratios to create six double-sorted imitating portfolios, following Fama and French (1993; 2015). To begin, all firms are separated equally into two categories called Small and Big portfolios. Second, these two portfolios are further subdivided into three categories (2 x 3 portfolios), which are shown in table 1 as Low, Medium, and High B|M ratios firms.

Table 1: Matrix of Portfolio Construction

<table>
<tr><th>Size</th><th>B|M Ratio</th><th>Portfolio</th><th>Name</th></tr>
<tr><td></td><td>Low B|M (L)</td><td>SL</td><td>SBM1</td></tr>
<tr><td>Small (S)</td><td>Med B|M (M)</td><td>SM</td><td>SBM2</td></tr>
<tr><td></td><td>High B|M (H)</td><td>SH</td><td>SBM3</td></tr>
<tr><td></td><td>Low B|M (L)</td><td>BL</td><td>SBM4</td></tr>
<tr><td>Big (B)</td><td>Med B|M (M)</td><td>BM</td><td>SBM5</td></tr>
<tr><td></td><td>High B|M (H)</td><td>BH</td><td>SBM6</td></tr>
<tr><td>Size</td><td>Small = (SL + SM + SH)/3</td><td>Big = (BL + BM + BH)/3</td><td></td></tr>
<tr><td>HML</td><td>High = (SH + BH)/2</td><td>Low = (SL + BL)/2</td><td></td></tr>
</table>

Notes: Table 1 depicts the portfolio creation matrix for six equally weighted portfolios. SL displays a portfolio of stocks with small market capitalizations and low B|M ratios. Similarly, SH displays a portfolio of stocks with small market capitalizations and high B|M ratios. Furthermore, BL displays a portfolio of stocks with large market capitalizations and low B|M ratios. Similarly, BH displays a portfolio of stocks with large market capitalizations and high B|M ratios. The last column depicts the formation of the SMB (size) and HML (value) anomalies.

Figure 1: Structural Equation Modeling using Liquidity as Mediator for FF5FM



Source: author’s compilation.

Notes: The figure 1 depicts the direct relationship between independent risk-premia and 6 portfolio stock returns and indirect relationship using Liquidity as mediator in this study.

Figure-1: SEM using Fama and French (2015) five-factor model including Liquidity as mediating variable impact on 6 excess portfolio returns on PSX.

Empirical Results and Discussions

This research investigates the models described in the preceding part to empirically examine the impact of various factors on various sets of portfolios that comprise of non-financial companies listed on the PSX from January 2010 to June 2022. This research retrieves all stock data from Thomson Reuters DataStream. In this study, the 3-month Treasury Bills rate is utilised as the risk-free rate, which is obtained from the official website of the State Bank of Pakistan. This study employs Fama and French (1993; 2015) dual sort procedure as the Size-B|M ratio to build 6 value-weighted portfolios.

Descriptive Statistics

The following table exhibits the descriptive statistics and correlation matrix of independent and mediating variables to be investigated:

Panel-A of Table 2 displays descriptive statistics for the independent variables market, size, value, profitability, investment, and liquidity (mediating variable). All factors, as illustrated, except market, have moderate mean returns and corresponding standard deviations. The market risk premium averages 0.002 percent and has a minimum fall of -0.46 percent, which is unfavourable for market returns. The HML-value factor, on the other hand, shows the second-lowest risk premium with a 28 percent reduction, while CMA (investment risk-premium) shows the highest returns with a (0.229) 23 percent incline. The other risk premiums exhibit normal distribution when it comes to the maximum and minimum values. Similar to this, the market shows the largest standard deviation (0.07) at 7 percent, while the liquidity risk premium shows the second-highest value (0.06) at 6 percent. As a consequence, the values in panel A seem to be normally distributed and the dataset does not contain any outliers.

The correlation matrix between the independent variables is presented in panel-B of Table 2. According to the findings, there is a strong correlation between value and market (0.2758), profitability and size (0.6535), investment and market (0.0256), liquidity and market (0.2694), and liquidity and value (0.6366) accordingly. Conversely, this group of couples exhibit negative nexus, as evidenced by size with market (-0.1859), value with size (-0.1964), profitability with market (-0.294) and with value (-0.1613), investment with size, value, and profitability showing (-0.644, -0.2518 and -0.4069), respectively, while liquidity with size, profitability, and investment exhibiting (-0.0134, -0.1345 and -0.2217), respectively.

Table 2: Descriptive statistics and correlation matrix

Panel-A	**Variable**	**RmRf**	**SMB**	**HML**	**RMW**	**CMA**	**IML**
	Mean	0.001905	-0.01035	-0.02839	-0.00521	0.022248	0.001756
	Std. Dev.	0.07011	0.041432	0.042553	0.030009	0.039639	0.055288
	Min	-0.45966	-0.1526	-0.27753	-0.09243	-0.1359	-0.18123
	Max	0.20034	0.101703	0.103502	0.143691	0.229755	0.111384
	Obs.	210	210	210	210	210	210
Panel-B	**Variable**	**RmRf**	**SMB**	**HML**	**RMW**	**CMA**	**IML**
	RmRf	1	-0.1859	0.2758	-0.294	0.0256	0.2694
	SMB	-0.1859	1	-0.1964	0.6535	-0.644	-0.0134
	HML	0.2758	-0.1964	1	-0.1613	-0.2518	0.6366
	RMW	-0.294	0.6535	-0.1613	1	-0.4069	-0.1345
	CMA	0.0256	-0.644	-0.2518	-0.4069	1	-0.2217
	IML	0.2694	-0.0134	0.6366	-0.1345	-0.2217	1

Notes: Table 2, Panel-A represents the descriptive statistics of all factors included in the study and mediating variable (liquidity) which consists of mean, standard deviation, maximum and minimum values. Panel-B shows correlation matrix among our test factors and mediating variable.

Structural Equation Model (SEM) Estimates

The following estimations were obtained using the SEM from Fama and French (2015) five-factor model with six equal-weighted portfolios:

Table 3: Structural Equation Model (SEM) Estimates

Endogenous	Exogenous	Coef.	z	P>|z|	Endogenous	Exogenous	Coef.	z	P>|z|
SBM1 <-	**IML**	0.9666	52.68	0.0000	**SBM4** <-	**IML**	0.9813	81.08	0.0000
	RmRf	0.0122	1.04	0.2960		**RmRf**	0.0138	1.79	0.0730
	SMB	0.3654	11.39	0.0000		**SMB**	-0.2666	-12.6	0.0000
	HML	0.3576	13.51	0.0000		**HML**	-0.0311	-1.78	0.0750
	RMW	-1.5386	-44.47	0.0000		**RMW**	0.1258	5.51	0.0000
	CMA	0.1835	6.35	0.0000		**CMA**	0.4035	21.18	0.0000
	_cons	-0.0021	-1.96	0.0500		**_cons**	-0.0001	-0.15	0.8830
	var(e.SBM1)	0.000119				**var(e.SBM4)**	0.000052		
SBM2 <-	**IML**	0.9892	30.29	0.0000	**SBM5** <-	**IML**	0.9146	27.51	0.0000
	RmRf	-0.0061	-0.29	0.7680		**RmRf**	0.0383	1.80	0.0710
	SMB	0.2303	4.03	0.0000		**SMB**	0.4641	7.99	0.0000
	HML	-0.0341	-0.72	0.4700		**HML**	0.6841	14.27	0.0000
	RMW	-0.2859	-4.64	0.0000		**RMW**	0.0486	0.78	0.4380
	CMA	0.371	7.22	0.0000		**CMA**	0.7705	14.73	0.0000
	_cons	0.0041	2.10	0.0360		**_cons**	-0.0043	-2.21	0.0270
	var(e.SBM2)	0.000378				**var(e.SBM5)**	0.000392		
SBM3 <-	**IML**	1.0187	84.17	0.0000	**SBM6** <-	**IML**	1.0847	34.80	0.0000
	RmRf	-0.0138	-1.79	0.0730		**RmRf**	-0.0488	-2.46	0.0140
	SMB	0.2666	12.60	0.0000		**SMB**	0.835	15.33	0.0000
	HML	0.0311	1.78	0.0750		**HML**	0.1585	3.53	0.0000
	RMW	-0.1258	-5.51	0.0000		**RMW**	-0.2165	-3.68	0.0000
	CMA	-0.4035	-21.18	0.0000		**CMA**	1.0148	20.69	0.0000
	_cons	0.0001	0.15	0.8830		**_cons**	-0.0037	-1.99	0.0470
	var(e.SBM3)	0.000052				**var(e.SBM6)**	0.000344		
IML <	**RmRf**	0.0727	1.66	0.0960					
	SMB	0.3486	2.95	0.0030					
	HML	0.8426	10.43	0.0000					
	RMW	-0.2865	-2.23	0.0260					
	CMA	0.0617	0.57	0.5700					
	_cons	0.0263	7.20	0.0000					

Notes: Table 3 presents the findings from a structural equation model that utilised the FF5FM, multidimensional liquidity as a mediating variable, and a six-value-weighted portfolio constructed in accordance with the Size-B|M ratio. The idiosyncratic risk factors are employed as exogenous variables, whilst portfolio stock returns and multidimensional liquidity are used as endogenous variables. Liquidity is used as mediating variable in this study. Coefficients, z-values, and related probability values are displayed in the findings.

The estimation results of risk-adjusted performance of 6 value-weighted portfolios, constructed based on Size-B|M ratio using the Fama and French (2015) five-factor model through Structural Equation Model (SEM) technique are reported in Table 3. The endogenous variables are the excess average portfolio returns (SBM1-SBM6) calculated depending on size and B|M ratio as indicated by (Fama & French, 1993; 2015). SBM6 is a portfolio that may alternatively be displayed as a BH portfolio. It consists of firms with large Market Capitalization (MC) and high B|M ratio average returns. However, the SBM1 portfolio is featured as an SL portfolio. It consists of firms having a low B|M return and small MC. Risk premiums are utilised as an exogenous variable to investigate the relationship between endogenous factors and a mediating variable such as multidimensional liquidity.

The results are categorized into two columns, the left hand-side shows portfolio SBM1, SBM2 and SBM3 and the right hand-side shows portfolio SBM4, SBM5 and SBM6 findings accordingly. The findings determine the direct nexus of liquidity risk premium with portfolio excess returns is highly statistically significant. On the other hand, the market risk premium exhibits statistically insignificant relationship with portfolio stock returns except one portfolio SBM6 (-0.0488) which shows significant coefficient. The market-factor show mix results in terms of magnitude, three positive (SBM1, SBM4, SBM5) and three negative (SBM2, SBM3, SBM6) which demonstrates inconsistent with the findings of (Azam, 2022c). For all portfolios, the size-factor produces highly statistically significant findings. Similarly, the value-factor presents as portfolios SBM1, SBM5 and SBM6 show highly significant findings while SBM2, SBM3 and SBM4 (-0.0341, 0.0311 and -0.0311 with probability values (0.470, 0.075 and 0.075) respectively demonstrates insignificant nexus with portfolio excess returns. The direct effect of the coefficients of profitability-factor demonstrate highly statistically significant findings except one portfolio (SBM5 = 0.0486) however, investment-factor exhibit highly statistically significant association with portfolio returns. The impact of risk-factors on mediating variable (liquidity) demonstrates statistically significant finding in the case of size, value and profitability (0.3486, 0.8426 and -0.2865) respectively. On the other hand, the coefficients of market and investment exhibit statistically insignificant impact on liquidity.

The results demonstrate that CAPM failed to forecast expected returns, leading to its discovery as redundant and a mispriced risk-factor, which is consistent with recent studies such as (Hasler, & Martineau, 2022; Azam, 2021; 2022c; Lohano & Kashif, 2018; Urooj & Shah, 2016). To infer, the monthly returns based on SEM framework show poor CAPM findings in PSX. In addition, PSX offers competitive prices for the remaining risk premiums, which include liquidity, size, profitability, and investment while value risk premium exhibits mix findings. Moreover, this study examines the mediating role of liquidity between risk-factors and portfolio returns and the direct nexus between liquidity augmented five-factors and portfolio stock returns. Conclusively, by employing the liquidity as mediating variable using SEM, we observed the following results:

Table 4: Conclusive estimates for SEM

Factors impact on Liquidity			**Factors and Liquidity impact on Portfolio Returns (SBM1-SBM6)**							
Factor	***P>|z|***	***Significance***	***Factor***	***SBM1***	***SBM2***	***SBM3***	***SBM4***	***SBM5***	***SBM6***	***Sig.***
RmRf	0.096	IS	IML	0.000	0.000	0.000	0.000	0.000	0.000	HS
SMB	0.003	HS	RmRf	0.296	0.768	0.073	0.073	0.071	0.014	IS
HML	0.000	HS	SMB	0.000	0.000	0.000	0.000	0.000	0.000	HS
RMW	0.026	HS	HML	0.000	0.470	0.075	0.075	0.000	0.000	MS
CMA	0.570	IS	RMW	0.000	0.000	0.000	0.000	0.438	0.000	HS
			CMA	0.000	0.000	0.000	0.000	0.000	0.000	HS

Notes: Table 4 displays the conclusive estimates for SEM with liquidity as mediating variable using FF5FM. IS= Insignificant, HS=highly Significant, MS=moderately Significant.

The results display in Table 4 demonstrates that factors such as size, value and profitability has statistically highly significant nexus with liquidity while including mediating variable i.e. liquidity and factors such as size, profitability and investment have statistically highly significant impact on portfolio returns proving the mediating role of liquidity in the model in PSX.

Time-Series OLS Regression Estimates

This study use the time-series OLS regression approach to further study the relationship between factors and average portfolio excess returns and to determine whether or not multidimensional liquidity as a risk factor yields statistically significant findings. The two models below are tested further for examining the performance and robustness of risk-factors impacting portfolio stock returns in PSX.

Fama and French (2015) five-factor model

The time-series OLS regression using FF5FM with six equal-weighted portfolios yielded the following estimates:

Table 5. Time-series OLS regression results for FF-5FM

VARIABLES	(1) **SBM1**	(2) **SBM2**	(3) **SBM3**	(4) **SBM4**	(5) **SBM5**	(6) **SBM6**
RmRf	0.083*	0.066	0.060	0.085*	0.105**	0.030
	(1.854)	(1.352)	(1.313)	(1.925)	(2.284)	(0.576)
SMB	0.702***	0.575***	0.622***	0.075	0.783***	1.213***
	(5.842)	(4.373)	(5.015)	(0.632)	(6.316)	(8.610)
HML	1.172***	0.799***	0.889***	0.796***	1.455***	1.072***
	(14.264)	(8.895)	(10.499)	(9.739)	(17.170)	(11.136)
RMW	-1.816***	-0.569***	-0.418***	-0.155	-0.213	-0.527***
	(-13.878)	(-3.979)	(-3.097)	(-1.194)	(-1.582)	(-3.439)
CMA	0.243**	0.432***	-0.341***	0.464***	0.827***	1.082***
	(2.203)	(3.579)	(-2.994)	(4.228)	(7.266)	(8.362)
Constant	0.023***	0.030***	0.027***	0.026***	0.020***	0.025***
	(6.269)	(7.397)	(7.019)	(6.955)	(5.142)	(5.707)
Observations	210	210	210	210	210	210
R-squared	0.737	0.367	0.574	0.445	0.631	0.438

Notes: The t-statistics are presented in parentheses with significance as *** p<0.01, ** p<0.05, * p<0.1. Table 5 presents time-series OLS regression estimates for FF5FM using time span (2010-2022).

The estimation outcomes of the five-factor model established by Fama and French (2015) using the time-series OLS regression approach are shown in Table 5. Average portfolio returns (SBM1-SBM6) created based on size and B|M ratio as proposed by (Fama & French, 1993; 2015) are the dependent variable for all six-OLS regressions. The portfolio that may also be presented as a BH portfolio is SBM6. It comprises of companies with big Market Capitalization (MC) and high B|M ratio firms average returns. The SBM1 portfolio may also be shown as an SL portfolio. It comprises of firms with small MC and low B|M returns.

The results demonstrate that market risk premium have positive but insignificant nexus with portfolio returns for portfolio (SBM2, SBM3 and SBM6) using FF5FM. Similarly, two portfolios (SBM1 and SBM4) show weak but significant nexus while one portfolio (SBM5) presents statistically moderate significant nexus with portfolio returns. The coefficient estimates of size shows positive and statistically significant magnitude (except SBM4) which shows insignificant association with portfolio stock returns. The beta coefficient values of value risk-premium display statistically highly significant and positive nexus with portfolio returns. However, the profitability risk-premium show highly significant but negative results except SBM4 and SBM5 (-0.155 and -0.213 respectively) shows show negative but insignificant relationship with portfolio excess-returns. Similarly, the coefficients of investment risk-premium demonstrates statistically highly significant but positive except one portfolio SBM3 (-0.341) which shows negative but highly significant nexus with portfolio excess returns. Growing-values are shown by the coefficient of determination (R^2) for both portfolios, with SBM1 ($R^2 = 0.737$) and SBM2 ($R^2 = 0.367$) having the greatest and lowest R-square values, respectively.

Based on the findings above, it is clear that CAPM cannot predict expected returns, hence modest premiums were assumed, which are consistent with recent research such as (Urooj & Shah, 2016; Lohano & Kashif, 2018). In conclusion, the monthly returns based on time-series OLS regression framework show poor CAPM in PSX due to the minimal impact of market returns on portfolio excess returns. In addition, PSX offers competitive prices for the remaining risk premiums, which include size, value, profitability, and investment (Fama and French, 2015).

Liquidity augmented Fama and French (2015) five-factor model

The time-series OLS regression using liquidity augmented FF5FM with six equal-weighted portfolios yielded the following estimates:

Table 6. Time-series OLS regression results for Liquidity augmented FF-5FM

	(1)	(2)	(3)	(4)	(5)	(6)
VARIABLES	**SBM1**	**SBM2**	**SBM3**	**SBM4**	**SBM5**	**SBM6**
RmRf	0.012	-0.006	-0.014*	0.014*	0.038*	-0.049**
	(1.026)	(-0.290)	(-1.760)	(1.760)	(1.774)	(-2.414)
SMB	0.365***	0.230***	0.267***	-0.267***	0.464***	0.835***
	(11.200)	(3.966)	(12.389)	(-12.389)	(7.854)	(15.071)
HML	0.358***	-0.034	0.031*	-0.031*	0.684***	0.158***
	(13.284)	(-0.711)	(1.753)	(-1.753)	(14.028)	(3.466)
RMW	-1.539***	-0.286***	-0.126***	0.126***	0.049	-0.217***
	(-43.718)	(-4.565)	(-5.421)	(5.421)	(0.762)	(-3.622)
CMA	0.184***	0.371***	-0.403***	0.403***	0.770***	1.015***
	(6.248)	(7.098)	(-20.828)	(20.828)	(14.482)	(20.343)
IML	0.967***	0.989***	1.019***	0.981***	0.915***	1.085***
	(51.793)	(29.778)	(82.752)	(79.715)	(27.052)	(34.218)
Constant	-0.002*	0.004**	0.000	-0.000	-0.004**	-0.004*
	(-1.924)	(2.065)	(0.145)	(-0.145)	(-2.177)	(-1.957)
Observations	210	210	210	210	210	210
R-squared	0.981	0.882	0.988	0.983	0.920	0.917

Notes: The t-statistics are presented in parentheses with significance as *** p<0.01, ** p<0.05, * p<0.1. Table 6 shows time-series OLS regression estimates for liquidity augmented FF5FM using time span (2010-2022).

In order to examine the relationship between market, size, value, profitability, investment, and liquidity on portfolio stock returns, Fama and French (2015) five-factor model is employed with multidimensional liquidity (Liu, 2006) as an augmented factor for further robustness by employing the OLS regression technique. The findings are remarkably similar to those of the FF-5FM-OLS regression but slightly better because the market risk-premium exhibit weak significant coefficients in four models (SBM3-SBM6) and only two portfolios exhibit insignificant coefficients (SBM1 and SBM2). With a probability of less than 0.01, the size, investment, and liquidity coefficients show highly statistically significant results shows similarity with (Azam, 2021). The value-premium indicates very significant outcomes for three portfolios (SBM1, SBM5, and SBM6), moderately significant results for two portfolios (SBM3 and SBM4), and negative and insignificant results for one portfolio (SBM2). The coefficient of determination (R-squared) reveals that liquidity performs better as an independent variable (risk-premium) in the PSX, with explanatory power ranging from 88 percent to 99 percent. Moreover, portfolio R-squared of SBM1 (74% using FF5FM increases to 98% using LFF5FM) indicates a 24% increase, whereas SBM2, SBM3, SBM4, SBM5, and SBM6 exhibit 52%, 41%, 54%, 29%, and 48% increases, respectively.

Conclusively, liquidity as mediating variable as well as risk-premium both explaining the portfolio returns in PSX. Thus, liquidity performs significant contribution as multidimensional characteristics which impact the stock/portfolio returns directly but also mediates between Fama and French five-factors and portfolio returns on PSX. Fama and French (2015) five-factor model is employed with (Liu, 2006) multidimensional liquidity as an augmented factor for further robustness by employing the OLS regression technique which can be presented in the following table:

Table 7. Conclusive estimates for OLS regression

FF5FM	β_M	β_S	β_V	β_P	β_I	LFF5FM	β_M	β_S	β_V	β_P	β_I	β_L
Significance	WS	S	HS	WS	HS	Significance	WS	HS	S	S	HS	HS
Total 6	3	5	6	4	6	Total 6	4	6	5	5	6	6

Notes: β represents Beta-coefficient, M=market, S = size, V=value, P=profitability, I=investment and L=liquidity. WS represents weakly significant, S= significant, and HS=highly significant. Table 7 shows the conclusive table for OLS regression estimates out of total 6 portfolios.

Table 7 demonstrates the conclusive results of OLS regression coefficients out of total 6 portfolios using FF5FM and LFF5FM using monthly data for PSX. The results conclude that market-factor exhibits weakly significant results in both the models while profitability-factor also shows weakly significant results in FF5FM. Moreover, all other-factors demonstrate highly statistically significant coefficients using OLS regression technique for PSX.

Conclusion

Conclusively, the Liu (2006) multidimensional liquidity is statistically significant in both situations: as mediating variable using SEM and as independent variable using OLS regression technique in PSX. The market risk-premium exhibits statistically insignificant results for PSX while size, profitability and investment also show significant findings in the market. The potential investors and portfolio managers need to consider liquidity as benchmark criteria prior to make decision regarding investing in PSX. Furthermore, using time-series OLS regression technique, the findings reveal that market-factor shows weakly significant outcomes in both FF5FM and L-FF5FM while profitability factor in FF5FM. However, other factors exhibit substantially significant findings for the OLS regression framework.

Furthermore, the study tests Liu (2006) multidimensional liquidity as mediating variable and as independent variable using SEM and OLS regression techniques, there is possibility to examine numerous proxies of liquidity as mediating variables for further robustness. As the liquidity cannot be measured by one model, therefore, researchers need to thoroughly study various proxies with the same methods to further investigate robust estimates using developed stock markets such as US, UK, Germany. The estimation results might be more appropriate in case more sophisticated econometrics techniques and tools will be applied. The sample size or multiple stock markets may produce more robust estimates. The Tobin-q risk premium can also be augmented with various nested models such as Fama and French (2015) using the same methods in developed stock markets.

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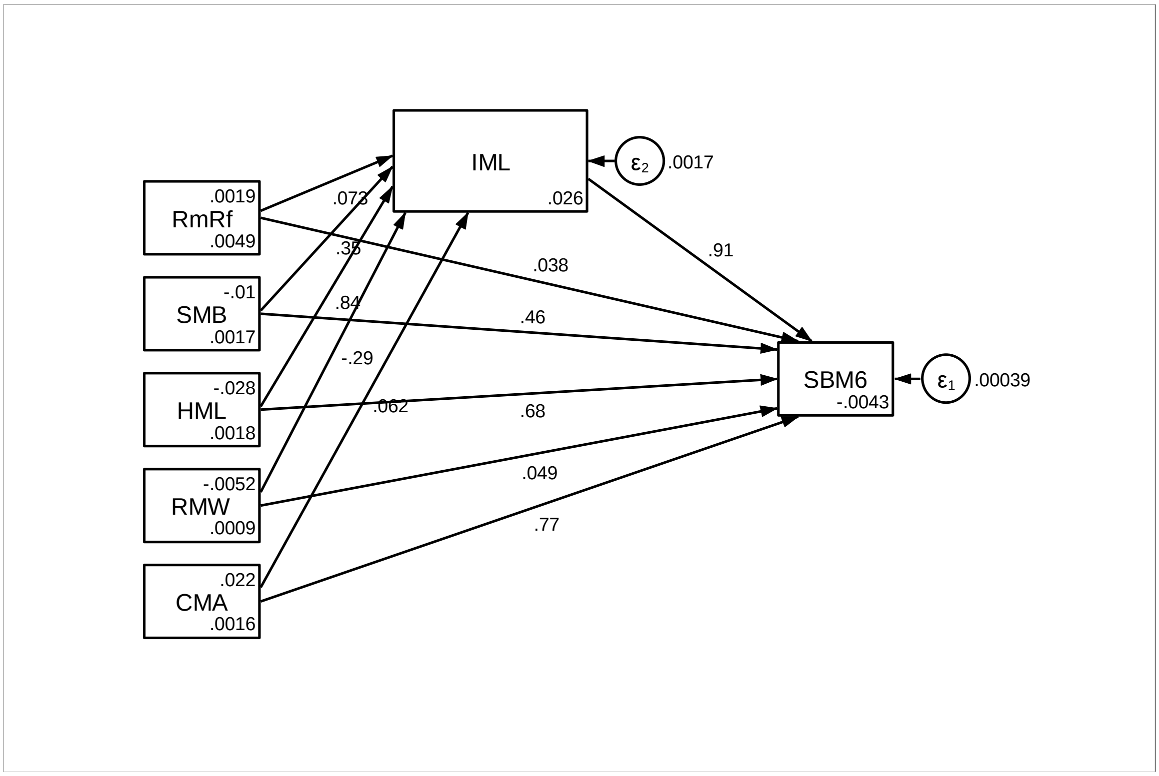
Appendix-A

Figure 2: Structural Equation Modeling (SEM)



Source: Author's Composition

Figure: Structural Equation Modeling (SEM) using Fama and French (2015) five-factor model with multidimensional liquidity (IML) of Liu (2006) as mediating variable in PSX. The five-factors are independent variables while equal-weighted six-portfolios are, constructed based on Size-B|M ratio, dependent variables of the study.

Appendix-B

Endogenous variables Observed: SBM1 IML

Exogenous variables Observed: RmRf SMB HML RMW CMA

Fitting target model:

Iteration 0: log likelihood = 3005.3004
Iteration 1: log likelihood = 3005.3004

Structural equation model Number of obs = 210
Estimation method = ml
Log likelihood = 3005.3004

```
------------------------------------------------------------------------------
             |                 OIM
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
Structural   |
  SBM1 <-    |
         IML |   .9666368   .0183499    52.68   0.000     .9306717    1.002602
        RmRf |   .0122225   .0117072     1.04   0.296    -.0107231    .0351682
         SMB |   .3653506   .0320732    11.39   0.000     .3024882     .428213
         HML |   .3575988   .0264673    13.51   0.000     .3057238    .4094738
         RMW |  -1.538614   .0346027   -44.47   0.000    -1.606434   -1.470794
         CMA |   .1835045   .0288762     6.35   0.000     .1269081    .2401008
       _cons |  -.0021219   .0010842    -1.96   0.050    -.0042469    3.18e-06
  -----------+----------------------------------------------------------------
  IML <-     |
        RmRf |    .072704   .0437392     1.66   0.096    -.0130232    .1584313
         SMB |   .3485865   .1181915     2.95   0.003     .1169354    .5802375
         HML |   .8426132   .0807828    10.43   0.000     .6842817    1.000945
         RMW |  -.2864994    .128616    -2.23   0.026    -.5385822   -.0344166
         CMA |   .0616509   .1085084     0.57   0.570    -.1510217    .2743235
       _cons |   .0262853   .0036517     7.20   0.000     .0191282    .0334425
-------------+----------------------------------------------------------------
  var(e.SBM1)|   .0001193   .0000116                      .0000986    .0001445
   var(e.IML)|   .0016875   .0001647                      .0013937    .0020432
------------------------------------------------------------------------------
LR test of model vs. saturated: chi2(0)   =      0.00, Prob > chi2 =      .
.
```

Endogenous variables Observed: SBM2 IML

Exogenous variables Observed: RmRf SMB HML RMW CMA

Fitting target model:

Iteration 0: log likelihood = 2884.2213
Iteration 1: log likelihood = 2884.2213

Structural equation model Number of obs = 210
Estimation method = ml
Log likelihood = 2884.2213

	Coef.	OIM Std. Err.	z	P>|z|	[95% Conf.	Interval]
Structural						
SBM2 <-						
IML	.9892065	.0326613	30.29	0.000	.9251915	1.053221
RmRf	-.0061388	.0208378	-0.29	0.768	-.0469802	.0347026
SMB	.2302745	.0570877	4.03	0.000	.1183845	.3421644
HML	-.0340509	.0471097	-0.72	0.470	-.1263842	.0582824
RMW	-.2859394	.0615899	-4.64	0.000	-.4066534	-.1652254
CMA	.3710467	.0513973	7.22	0.000	.2703099	.4717836
_cons	.0040533	.0019298	2.10	0.036	.0002709	.0078357
IML <-						
RmRf	.072704	.0437392	1.66	0.096	-.0130232	.1584313
SMB	.3485865	.1181915	2.95	0.003	.1169354	.5802375
HML	.8426132	.0807828	10.43	0.000	.6842817	1.000945
RMW	-.2864994	.128616	-2.23	0.026	-.5385822	-.0344166
CMA	.0616509	.1085084	0.57	0.570	-.1510217	.2743235
_cons	.0262853	.0036517	7.20	0.000	.0191282	.0334425
var(e.SBM2)	.000378	.0000369			.0003122	.0004577
var(e.IML)	.0016875	.0001647			.0013937	.0020432

LR test of model vs. saturated: chi2(0) = 0.00, Prob > chi2 = .
.

```
Endogenous variables Observed:  SBM3 IML

Exogenous variables Observed:  RmRf SMB HML RMW CMA

Fitting target model:

Iteration 0:   log likelihood =  3092.6922
Iteration 1:   log likelihood =  3092.6922

Structural equation model                     Number of obs     =        210
Estimation method  = ml
Log likelihood     =  3092.6922

------------------------------------------------------------------------------
             |                 OIM
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
Structural   |
  SBM3 <-    |
         IML |   1.018693   .0121032    84.17   0.000     .9949712    1.042415
        RmRf |  -.0138208   .0077218    -1.79   0.073    -.0289553    .0013137
         SMB |    .266574   .0211549    12.60   0.000     .2251112    .3080368
         HML |   .0311266   .0174573     1.78   0.075    -.0030891    .0653424
         RMW |  -.1258352   .0228233    -5.51   0.000     -.170568   -.0811025
         CMA |  -.4034666   .0190462   -21.18   0.000    -.4407964   -.3661367
       _cons |   .0001052   .0007151     0.15   0.883    -.0012964    .0015069
  -----------+----------------------------------------------------------------
  IML <-     |
        RmRf |    .072704   .0437392     1.66   0.096    -.0130232    .1584313
         SMB |   .3485865   .1181915     2.95   0.003     .1169354    .5802375
         HML |   .8426132   .0807828    10.43   0.000     .6842817    1.000945
         RMW |  -.2864994    .128616    -2.23   0.026    -.5385822   -.0344166
         CMA |   .0616509   .1085084     0.57   0.570    -.1510217    .2743235
       _cons |   .0262853   .0036517     7.20   0.000     .0191282    .0334425
-------------+----------------------------------------------------------------
  var(e.SBM3)|   .0000519   5.07e-06                      .0000429    .0000629
   var(e.IML)|   .0016875   .0001647                      .0013937    .0020432
------------------------------------------------------------------------------
LR test of model vs. saturated: chi2(0)   =      0.00, Prob > chi2 =      .
.
```

Endogenous variables Observed: SBM4 IML

Exogenous variables Observed: RmRf SMB HML RMW CMA

Fitting target model:

Iteration 0: log likelihood = 3092.6922
Iteration 1: log likelihood = 3092.6922

```
Structural equation model                  Number of obs     =       210
Estimation method  = ml
Log likelihood     =  3092.6922

------------------------------------------------------------------------------
             |                 OIM
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
Structural   |
  SBM4 <-    |
         IML |   .9813069   .0121032    81.08   0.000      .957585    1.005029
        RmRf |   .0138208   .0077218     1.79   0.073    -.0013137    .0289553
         SMB |   -.266574   .0211549   -12.60   0.000    -.3080369   -.2251112
         HML |  -.0311266   .0174573    -1.78   0.075    -.0653424    .0030891
         RMW |   .1258352   .0228233     5.51   0.000     .0811024     .170568
         CMA |   .4034665   .0190462    21.18   0.000     .3661367    .4407964
       _cons |  -.0001052   .0007151    -0.15   0.883    -.0015069    .0012964
  -----------+----------------------------------------------------------------
  IML <-     |
        RmRf |    .072704   .0437392     1.66   0.096    -.0130232    .1584313
         SMB |   .3485865   .1181915     2.95   0.003     .1169354    .5802375
         HML |   .8426132   .0807828    10.43   0.000     .6842817    1.000945
         RMW |  -.2864994    .128616    -2.23   0.026    -.5385822   -.0344166
         CMA |   .0616509   .1085084     0.57   0.570    -.1510217    .2743235
       _cons |   .0262853   .0036517     7.20   0.000     .0191282    .0334425
-------------+----------------------------------------------------------------
 var(e.SBM4)|   .0000519   5.07e-06                       .0000429    .0000629
  var(e.IML)|   .0016875   .0001647                       .0013937    .0020432
------------------------------------------------------------------------------
LR test of model vs. saturated: chi2(0)   =      0.00, Prob > chi2 =      .
.
```

Endogenous variables Observed: SBM5 IML

Exogenous variables Observed: RmRf SMB HML RMW CMA

Fitting target model:

Iteration 0: log likelihood = 2880.5338
Iteration 1: log likelihood = 2880.5338

```
Structural equation model                Number of obs     =        210
Estimation method  = ml
Log likelihood     =  2880.5338

------------------------------------------------------------------------------
             |                 OIM
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
Structural   |
  SBM5 <-    |
         IML |   .9145806   .0332399    27.51   0.000     .8494316    .9797296
        RmRf |   .0382659    .021207     1.80   0.071     -.003299    .0798308
         SMB |   .4641272    .058099     7.99   0.000     .3502551    .5779992
         HML |   .6840709   .0479442    14.27   0.000      .590102    .7780399
         RMW |   .0486068    .062681     0.78   0.438    -.0742457    .1714592
         CMA |   .7704755   .0523078    14.73   0.000     .6679541    .8729968
       _cons |   -.004349    .001964    -2.21   0.027    -.0081984   -.0004995
  -----------+----------------------------------------------------------------
  IML <-     |
        RmRf |    .072704   .0437392     1.66   0.096    -.0130232    .1584313
         SMB |   .3485865   .1181915     2.95   0.003     .1169354    .5802375
         HML |   .8426132   .0807828    10.43   0.000     .6842817    1.000945
         RMW |  -.2864994    .128616    -2.23   0.026    -.5385822   -.0344166
         CMA |   .0616509   .1085084     0.57   0.570    -.1510217    .2743235
       _cons |   .0262853   .0036517     7.20   0.000     .0191282    .0334425
-------------+----------------------------------------------------------------
 var(e.SBM5)|   .0003915   .0000382                      .0003234    .0004741
  var(e.IML)|   .0016875   .0001647                      .0013937    .0020432
------------------------------------------------------------------------------
LR test of model vs. saturated: chi2(0)   =      0.00, Prob > chi2 =      .
.
```

Endogenous variables Observed: SBM6 IML

Exogenous variables Observed: RmRf SMB HML RMW CMA

Fitting target model:

Iteration 0:   log likelihood = 2894.0611
Iteration 1:   log likelihood = 2894.0611

Structural equation model   Number of obs   =   210
Estimation method = ml
Log likelihood   = 2894.0611

```
------------------------------------------------------------------------------
             |                 OIM
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
Structural   |
  SBM6 <-    |
         IML |    1.08469   .0311662    34.80   0.000     1.023606    1.145775
        RmRf |  -.0488279    .019884    -2.46   0.014    -.0877998    -.009856
         SMB |   .8350074   .0544745    15.33   0.000     .7282393    .9417755
         HML |   .1584615   .0449532     3.53   0.000     .0703549    .2465682
         RMW |  -.2165084   .0587706    -3.68   0.000    -.3316967   -.1013201
         CMA |    1.01478   .0490446    20.69   0.000      .918654    1.110905
       _cons |  -.0036655   .0018415    -1.99   0.047    -.0072748   -.0000563
  -----------+----------------------------------------------------------------
  IML <-     |
        RmRf |    .072704   .0437392     1.66   0.096    -.0130232    .1584313
         SMB |   .3485865   .1181915     2.95   0.003     .1169354    .5802375
         HML |   .8426132   .0807828    10.43   0.000     .6842817    1.000945
         RMW |  -.2864994    .128616    -2.23   0.026    -.5385822   -.0344166
         CMA |   .0616509   .1085084     0.57   0.570    -.1510217    .2743235
       _cons |   .0262853   .0036517     7.20   0.000     .0191282    .0334425
-------------+----------------------------------------------------------------
 var(e.SBM6)|   .0003442   .0000336                      .0002843    .0004168
  var(e.IML)|   .0016875   .0001647                      .0013937    .0020432
------------------------------------------------------------------------------
```

LR test of model vs. saturated: chi2(0)   =   0.00, Prob > chi2 =   .

.

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