

RESEARCH ARTICLE

Unveiling the black swan of the Real Risk>Returns Nexus: Evidence from Pakistan Stock Exchange

Mohammad Azam¹, Ghlama Haddad², Naveed Naveed³, Arshad Iqbal⁴

¹Department of Management Sciences, National University of Modern Languages, Peshawar Campus, Pakistan

²Higher institute of Management, LaREMFIQ, University of Sousse, Tunisie

³Qurtuba University of Science & Information Technology, Peshawar Campus, Pakistan

⁴Brains Post Graduate College, Peshawar, Pakistan

Corresponding author: Mohammad Azam; email: azam.ims@hotmail.com

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Abstract

The risk-free rates are widely used as benchmark to measure excess stocks returns or excess market returns and contribute a significant role in Asset Pricing Models. The purpose of this study is to scrutinize the risk and real excess portfolio returns using inflation adjusted risk-free rates, a unique measuring technique with a primary focus on the momentum augmented Fama-French five-factor model, utilising monthly data for 1994-2022 from the Pakistan Stock Exchange. Using OLS regression technique, the findings reveal that except profitability, the market, size, value, momentum and investment move largely correlated with excess portfolio stocks returns. The Gibbons, Ross & Shanken test confirms that the momentum augmented Fama-French five-factor model outperforms in the market.

Keywords: Real-excess portfolio returns; Momentum augmented six-factor model; Asset Pricing Models; GRS test; Pakistan Stock Exchange

Introduction

The assessment of prices in financial markets is the dominating field of applied economics. Thus, it is susceptible to demand and supply in stock markets. The significance of the stock market cannot be ignored in the study of finance and economics which contributes a major part in the economy of a country (Umar, Ji, Mirza, & Rahat, 2021). In micro-perspective, it smoothen the process of fund-flow from savers to investors and makes their decision-making process convenient regarding investment and financing activities. The decision of investors while investing in stock market securities make them hyperconscious due to higher associated risk which compel them to utilize better tools and techniques for evaluating equity securities before constructing portfolios with the intention to diversify and hedge for beating the market and earning excess realized gains (Majeed, & Yan, 2022). Similarly, investors assume a standard benchmark for comparing the excess yields from their investment. Moreover, practitioners and portfolio managers consider risk-free rate as benchmark for comparing the excess portfolio and market returns.

The risk-free rates widely used as benchmark to measure excess stocks returns or excess market returns contribute a significant role in Asset Pricing Models (APMs). The

practitioners and academicians used multiple proxies of short or long term Government securities (Mukherji, 2011) such as Government bond market provides base for originating yields of other financial securities as standard and also assumes as proxy for risk-free investment (Rathnasingha & Dayarathne, 2021). Moreover, the CAPM equilibrium model is based on the Government treasury bills rates which assume surplus returns over risk-free rates. Therefore, the APMs postulate as baseline standard for measuring excess returns.

Since last few decades, there are various theoretical and empirical justified APMs revealed that these APMs contribute statistically significant nexus between factors (risk-premiums) and average portfolio stock returns in micro-level while imperative support to the economic development in macro-level, in order to explain the mechanism of stock prices determination and flow-of-funds respectively in developed and emerging stock markets.

In macro-perspective, the prior literature highlighted the substantial resulting and progressive nexus between asset pricing models (APMs) and economic development. As, the APMs evaluate stock prices which in response accelerate the decision-making process and confidence of investors in the stock market to convert savings into investments (Weston, 1973) which strengthen the

financial sector development that is a significant component of economic development (Umaret al., 2020; Umar et al., 2021). Besides, financial sector development (FSD) ultimately influence the economic growth, while conversely reduces the chances of financial crisis (Suet al., 2021; Suet al., 2020; Umar et al., 2021). Moreover, FSD proliferates investments in the economy which improve productivity, similarly, increases purchasing power and prosperity that conversely alleviates poverty (Jianget al., 2021).

Lower (rather than negative) inflation diminishes the threat of economic recession by allowing the labour market to respond more quickly during a downturn, and it also mitigates the risk that monetary policy may fail to stabilise the economy due to a liquidity-trap. Due to low inflation, the real salaries would be higher, as well as the savings. As a result, this explains why economists prefer a low and stable inflation rate presently. It will encourage investment, boost exports and avert an economic boom. On the negative side, it causes low aggregate demand and economic growth, as well as the potential risk of recession, high unemployment and slows production (Hong, 2021).

The economic theory demonstrates that inflation influences the investment mechanism and capabilities of investors. Siegel and Thaler (1997) argued that Fisher (1930), the pioneer of neoclassical economics, proposed the theory of interest which put down foundation for modern theory of intertemporal choice which explains how the current decisions influence by the potential opportunities or substitutions accessible in the future. There are four features associated with individual income such as its magnitude, time, consumption and risk. Conversely, inflation is a curse which impacts adversely the income of individuals. The individual investors expect the returns from their investments regardless of inflation factors which infer inappropriate and what Fisher termed real returns.

In fact, in emerging economies such as Pakistan, where about 35 percent of the population lives on less than \$2 per day, inflation may be a double-edged sword, causing investors to fall into poverty (Idrees & Baig, 2017; Ullahet al., 2020) such as 26.10% is recorded for Jul-Aug 2021-2022¹ which is an alarming situation for capital market investors and portfolio managers which need to evaluate from the inflation perspectives which demonstrates the research gap to be considered in emerging economy of Pakistan.

Therefore, this study assumes inflation adjusted expected real returns in order to explore the nexus between various risk-premiums and real stock returns after eradicating the inflation risk. In the horserace of anomalies, such as

macroeconomic variables, labour income and future consumption, this study endeavor to assume real excess returns in merging stock market. Based on the background information, this study focuses on the following research objectives: Firstly, to examine the impact of inflation-adjusted as risk-free rates in APMs using emerging equity market data, then, to compare various APMs and factors including inflation-adjusted market, size, value, momentum, profitability and investment risk premiums in PSX and finally to find out the most appropriate asset pricing model among various APMs using GRS test.

Literature Review

A plethora of prior literature has underlined the inflation rate inverse nexus with stock returns. Similarly, Stone (1974) in his two-factor model proposed change in interest rate augmented CAPM. Jareño (2008) proposed an alternative five-factor model by augmenting changes in real interest rates and shocks in expected inflation with FF3FM.

Over the past decades, the APMs have experienced a tremendous growth in academic research. Therefore a plethora of theoretical and empirical research studies has been investigated various augmented anomalies to Capital Asset Pricing Model (CAPM), sometimes called market model (Ewald et al., 2021; Haddad & Hellara, 2019), and has been buttressed to investigate the efficient predictability explanatory power of the specification. Among these studies (Banz, 1981) proposed size pattern (Bhandari, 1988) leverage pattern (Haddad & Hellara, 2019) liquidity-augmented model (Basu, 1983) earnings to price ratio (Fama & French, 1993) jointly recommended size and B|M as anomalies of CAPM which was later on recognized as benchmark for investors and portfolio managers around the globe. Fama and French (2016) argued that FF3FM describes all anomalies but not momentum. Carhart (1997) used another anomaly as momentum which was augmented with FF3FM and became familiar as (Carhart, 1997) four-factor model (C4FM). Carhart claimed that momentum factor augmented with FF3FM comparatively proved successfully its contribution to the explanatory power of the model. After long time, Fama and French (2015) proposed further two-anomalies named profitability and investment which is thoroughly investigated but still there no convincing conclusion regarding profitability and investment patters performance particularly in emerging equity market. Fama & French (2018) claimed value-factor redundancy when simultaneously regressed with profitability and investment.

In financial economics, the asset pricing models (APMs) predominantly assume risk-free rate as benchmark to measure excess stock, portfolio and market returns based

¹https://www.pbs.gov.pk/sites/default/files/press_releases/2022/CPI_Press_Release_September_2022.pdf

on theories support the Capital Asset Pricing Model (CAPM) since 1960s. Moreover, the CAPM also supports to recognize risk-premia well (Ewaldet al., 2021). Although, the substantial empirical studies using excess portfolio returns using stock returns minus risk-free rate (mostly assumed as Government Treasury Bills rates) with the assumption that investors will get excess over risk-free. However, the economic theory supports that Treasury bills rate includes inflation plus time-value of money (Goetzmann et al., 2014). This study considers real stock returns, therefore, deducts inflation rate from average stock returns by following the theory of real interest rate to investigate whether adjusting for inflation the asset pricing models (APMs) yield statistically and economically substantial excess returns in emerging market of Pakistan.

There is a plethora of studies relied extensively on inflation as independent variable and regressed with stock returns to examine its long and short term association in various equity markets. Inflation has long-run inverse nexus with equity returns (Geetha et al., 2011; Saleem et al., 2013; Shahet al., 2020; Tripathi & Kumar, 2014). Mukherji (2011) examined the risk-free rates of CAPM using multiple proxies. The short and long-term T-bills and bonds are scrutinized using monthly mean real returns and inflation risks US data for 1926-2007. The S&P 500 index is used as market real returns. Moreover, they used inflation and market returns as independent variables while real stocks return as dependent variable. The short, intermediate and long-term Government securities are used for analysis. Their results reveal that T-bills are not having market risk for one and five-year duration using univariate and multivariate regressions. Treasury securities, convincingly, entail significant inflation risk.

Choice of the Risk-Free Rate

A number of studies have used various proxies like risk-free rates to analyse APMs around the world, as follows:

Table 1: Proxy used for Risk-free rate

| Author(s) | Proxy for Risk-free rate |
|---|---|
| Ansari (2000) | Commercial Banks’ term deposits Interest Rate. |
| Deb, Banerjee, and Chakrabarti (2007) | Govt. Securities having more than 5 years maturity. |
| Dilidüzgün, Yılmaz, and Selçuk (2020) | Government securities of 10-years maturity yields. |
| Prathama, Sugiarto, Ugut, and Hulu (2020) | Govt. 10 years Zero coupon bonds’ yields. |
| Stowe, Robinson, Pinto, and McLeavey (2007) | Govt. bonds having 10-20 years maturity yields. |
| Fama and French (1993, 2015) | Govt. Treasury Bills Rates (Short-term). |

Table 1 demonstrates the proxy used as risk-free rate around the globe. Although, in CAPM theory, the treasury-bills rates are assumed as risk-free returns (as benchmark) but investors yield adversely influenced by the inflation that is a common phenomenon exists everywhere which investors need to be compensated therefore. This study assumes the inflation rates as risk-free rate.

Methodology and Models Specification

Methodology

It has been critically claimed that a number of remarkable empirical APMs are inadequate since they do not create portfolios based on size and B|M ratio (Lewellen, Nagel, & Shanken, 2010). They further argued that higher R-square and low pricing errors are not strong supportive evidences to be concluded regarding the standard model. Therefore, this study constructed 25 mimicking portfolios as suggested by (Fama & French, 1993; 2015; Azam, 2021) based on market-cap and B|M ratio using 521 financial and non-financial firms enlisted on PSX. Moreover, coefficients and associated t-statistics significance are assumed to explain the nexus with portfolio average stocks returns in the market.

Models Specification:

This study empirically investigates various APMs using the following measurements specifications:

Mimicking Size-B|M ratio Stocks Portfolio Returns

The returns from stocks are calculated for each stock:

$$R_{pt} = \ln(P_t/P_{t-1}) \quad (1)$$

Where, R_{pt} is the portfolio stocks returns, p indicates portfolio, t indicates time (months). \ln is natural log, P_t is Closing price of this month and P_{t-1} is the Closing price of previous month.

Capital Asset Pricing Model (CAPM)

$$R_{pt} - Inf_{CPI} = \alpha + \beta_m(R_m - Inf_{CPI}) + \epsilon_i \quad (2)$$

In which, R_{pt} is expected excess return from portfolio. Inf_{CPI} , is Inflation rate (used as proxy). R_m is the expected return from market. β_m is the sensitivities or factor loading of market factor.

Table 2: 25 Equally-weighted Portfolios constructed based on Size and B|M Ratio by following (Fama & French, 1993; 2015)

| PF | L_B M | 2 | 3 | 4 | H_B M | PF | L_B M | 2 | 3 | 4 | H_B M |
|--------------|-------|----|----|----|-------|--------------|-------|-------|-------|-------|-------|
| Small | SL | S2 | S3 | S4 | SH | Small | SBM1 | SBM2 | SBM3 | SBM4 | SBM5 |
| 2 | 2L | 22 | 23 | 24 | 2H | 2 | SBM6 | SBM7 | SBM8 | SBM9 | SBM10 |
| 3 | 3L | 32 | 33 | 34 | 3H | 3 | SBM11 | SBM12 | SBM13 | SBM14 | SBM15 |
| 4 | 4L | 42 | 43 | 44 | 4H | 4 | SBM16 | SBM17 | SBM18 | SBM19 | SBM20 |
| Big | BL | B2 | B3 | B4 | BH | Big | SBM21 | SBM22 | SBM23 | SBM24 | SBM25 |

Notes: Table 2 shows equally-weighted 25 excess portfolios which are constructed based on size-B|M ratio. SL denotes the Small size-stocks and Low B|M ratio stocks, similarly BH denotes the Big size-stocks and High B|M ratio stocks portfolios. On the right hand, the names of these portfolio such as Size and B|M ratio 01, and so on.

Fama & French (1993) three-factor model (henceforth FF3FM)

$$R_{pt} - Inf_{CPI} = \alpha + \beta_m(R_m - Inf_{CPI}) + \beta_s(SmB) + \beta_v(HmL) + \varepsilon_i(3)$$

In which, (SmB) is the small market-cap stocks portfolio minus big market-cap stocks portfolios. Similarly, (HmL) is the value firms (having high B|M ratio stocks portfolio) minus growth firms (having low B|M ratio portfolio). β_m , β_s , and β_v , are the coefficients of market, size and value factors respectively.

Carhart (1997) four-factor model (henceforth C4FM)

$$R_{pt} - R_f = R_f + \beta_m(R_m - R_f) + \beta_s(SmB) + \beta_v(HmL) + \beta_w(WmL) + \varepsilon_i(4)$$

In which, WML is the Winner (portfolios having positive higher returns for lag 2 years) minus Losers (portfolios having positive higher returns for lag 2 years' firms returns) known as Momentum factor. β_w is the coefficients of momentum factor. Momentum factor is augmented with FF3FM as equation (3).

Fama & French (2015) five-factor model (henceforth FF5FM)

$$R_{pt} - Inf_{CPI} = \alpha + \beta_m(R_m - Inf_{CPI}) + \beta_s(SmB) + \beta_v(HmL) + \beta_p(RmW) + \beta_i(CmA) + \varepsilon_i(5)$$

In which, (RmW) is the stocks having robust profitability portfolio minus stocks having weak profitability portfolio. Similarly, (CmA) is the conservative stocks portfolio (having higher assets) minus aggressive stocks portfolio (having lower assets). β_p and β_i , are the coefficients of profitability and investment factors respectively.

Momentum augmented Fama & French (2015) five-factor model (henceforth M-FF5FM)

$$R_{pt} - Inf_{CPI} = \alpha + \beta_m(R_m - Inf_{CPI}) + \beta_s(SmB) + \beta_v(HmL) + \beta_p(RmW) + \beta_i(CmA) + \beta_w(WmL) + \varepsilon_i(6)$$

GRS (Gibbons, Ross & Shanken, 1989) Test

$$GRS = \left(\frac{T}{N}\right) \left(\frac{T-N-L}{T-L-1}\right) \left[\frac{\hat{\alpha}' \hat{\Sigma}^{-1}}{1 + \hat{\mu}' \hat{\Omega}^{-1} \hat{\mu}} \right] \sim F(N, T - N - L) \quad (7)$$

Where,

$\hat{\alpha}$ = N x 1 estimated constant term vector.

$\hat{\Sigma}$ = Stochastic terms unbiased covariance matrix.

$\hat{\mu}$ = L x 1 factor portfolio average matrix.

$\hat{\Omega}$ = Factor portfolio unbiased covariance matrix.

T = No. of observations.

N = No. of regression equations.

L = No. of factors in the regression.

Using the above equation, this study examines the GRS-F test based on the following hypothesis: $H_0: \alpha_i = 0$ i: 1, 2, 3... N, where, the GRS-F test denotes that all alpha coefficients are equal to zero ($\alpha=0$) while $H_1: \alpha_i \neq 0$ i: 1, 2, 3... N, where, the GRS-F test denotes that all alpha coefficients are not equal to zero ($\alpha \neq 0$).

Gibbons, Ross, and Shanken (1989) employed this specification with a view to investigate the variations in the intercepts of portfolios. It measures the conditional efficiency of a particular portfolio based on the risk-free rates if the risk-free rates variate positively. It further examines the authentic and appropriately explaining specification based on absolutely average alpha (AAA) among various APMS.

Descriptive Statistics

Table 3: Descriptive Statistics of 25 Value-weighted Portfolios constructed based on dual-sorted Size and B|M Ratio by following (Fama & French, 1993; 2015; Azam, 2021)

| Mean | L_B M | 2 | 3 | 4 | H_B M | Max | L_B M | 2 | 3 | 4 | H_B M |
|--------------|---------|---------|---------|---------|---------|--------------|--------|--------|--------|--------|--------|
| Small | -0.0192 | -0.0176 | -0.0170 | -0.0177 | -0.0225 | Small | 0.257 | 0.295 | 0.312 | 0.263 | 0.309 |
| 2 | -0.0162 | -0.0156 | -0.0115 | -0.0158 | -0.0207 | 2 | 0.407 | 0.249 | 0.243 | 0.268 | 0.248 |
| 3 | -0.0181 | -0.0124 | -0.0125 | -0.0146 | -0.0118 | 3 | 0.260 | 0.367 | 0.188 | 0.225 | 0.246 |
| 4 | -0.0173 | -0.0148 | -0.0148 | -0.0127 | -0.0124 | 4 | 0.187 | 0.189 | 0.157 | 0.226 | 0.214 |
| Big | -0.0166 | -0.0149 | -0.0122 | -0.0108 | -0.0108 | Big | 0.182 | 0.169 | 0.176 | 0.337 | 0.219 |
| Std. Dev. | L_B M | 2 | 3 | 4 | H_B M | Min | L_B M | 2 | 3 | 4 | H_B M |
| Small | 0.065 | 0.075 | 0.068 | 0.082 | 0.067 | Small | -0.208 | -0.353 | -0.270 | -0.277 | -0.271 |
| 2 | 0.077 | 0.069 | 0.071 | 0.072 | 0.072 | 2 | -0.324 | -0.233 | -0.227 | -0.341 | -0.263 |
| 3 | 0.071 | 0.069 | 0.062 | 0.065 | 0.063 | 3 | -0.332 | -0.237 | -0.249 | -0.201 | -0.217 |
| 4 | 0.065 | 0.063 | 0.061 | 0.069 | 0.067 | 4 | -0.286 | -0.233 | -0.200 | -0.318 | -0.263 |
| Big | 0.077 | 0.065 | 0.059 | 0.069 | 0.065 | Big | -0.421 | -0.367 | -0.392 | -0.319 | -0.249 |

Notes: Table 1 shows the descriptive statistics for 25 value-weighted inflation adjusted 25 excess portfolios which are constructed based on Size and B|M ratio following Fama & French (1994; 2015). It consists of average monthly returns, standard deviation (SD), Maximum and minimum excess portfolio returns.

Table 3 illustrates the descriptive statistics of 25 value-weighted inflation adjusted portfolios (returns minus inflation) using monthly data from Jan-1994 through Dec-2022. The small market-cap and high B|M ratio excess portfolio demonstrates -0.0225 average real returns with standard deviation of 0.067 while the big market-cap and low B|M ratio excess portfolio demonstrates -0.0166 average real returns with standard deviation of 0.077. Although, Table-1 shows negative average portfolios returns but inversely, small market-cap firms and high B|M ratio portfolio yield lower real returns (-0.0225) and portfolio consists of big market-cap and low B|M ratio yield higher excess real returns (-0.0166). On the other hand, small market-cap firms and low B|M ratio portfolio yield lower real returns (-0.0192) with standard deviation of (0.065) and portfolio consists of big market-cap and high B|M ratio yield higher excess real returns (-0.0108

with standard deviation of 0.065). On average, the small 10 portfolios exhibit -0.01738 excess real returns with average standard deviation of 0.0718 while the big 10 portfolios exhibit -0.01373 excess real returns with average standard deviation of 0.066. Theoretically, the portfolio of small market-cap firms should yield more returns than big market-cap firms but the findings demonstrates contradictory results as big market-cap shows negative but higher returns. Moreover, on average, the 10 portfolios consist of Low-B|M ratios exhibit -0.01627 returns having standard deviation of 0.0696 while the High-B|M ratio exhibit -0.01498 returns having standard deviation of 0.0691 which exhibit theoretically supportive results as high B|M ratio (value) firms outperform low B|M ratio (growth) firms.

Table 4: Correlation Matrix

| Factor | RmInf | SMB | HML | RMW | CMA | WML |
|--------------|---------|--------|---------|--------|--------|-----|
| RmInf | 1 | | | | | |
| SMB | -0.0475 | 1 | | | | |
| HML | 0.025 | 0.0423 | 1 | | | |
| RMW | 0.0154 | -0.022 | -0.2532 | 1 | | |
| CMA | -0.0034 | 0.077 | 0.2495 | 0.1168 | 1 | |
| WML | -0.0935 | 0.2302 | -0.2486 | 0.0906 | 0.2692 | 1 |

Notes: Table 4 summarizes the correlation matrix between independent variables including market-inflation adjusted premium, size-premium, value-premium, profitability-premium, investment premium and momentum premium.

Table 4 demonstrates the correlation matrix between independent (RHS) variables. Grobys and Huhta-Halkola (2019) argue that inverse correlation between two patterns have the potential of yielding higher expected returns by

constructing diversified portfolio. There exists inverse correlation between inflation adjusted market returns and size pattern, investment and momentum. Similarly, size with profitability; value with profitability and momentum

also demonstrate negative nexus between each other. As the maximum value shows between investment and momentum pattern (0.2692) but positive which indicates that there is no chance of multicollinearity problem while employing time-series OLS regression.

Average Annual Returns for Factors (Independent Variables)

Table 5 demonstrates the inflation adjusted market, size, value, profitability, investment and momentum annual returns based on each year.

Table 5: Annual inflation-adjusted market and other factors average returns:

| Year | RmInf | SMB | HML | RMW | CMA | WML |
|------|----------|----------|----------|----------|----------|----------|
| 1994 | 0.042269 | -0.00277 | -0.00019 | -0.00038 | 0.000227 | -0.0001 |
| 1995 | -0.01449 | -0.00137 | -0.00304 | 0.002521 | -8.4E-05 | -0.00046 |
| 1996 | -0.03179 | 0.006434 | -0.00126 | 0.000613 | -0.00028 | 0.007555 |
| 1997 | -0.01475 | -0.00011 | 0.000316 | 0.001458 | -0.00062 | 0.007773 |
| 1998 | 0.0358 | 0.00608 | 0.000883 | 0.00164 | -0.00065 | 0.002219 |
| 1999 | -0.03389 | 0.001592 | -0.00054 | 0.001826 | -0.00107 | 0.009978 |
| 2000 | 0.014341 | -0.00181 | -0.00221 | 0.002513 | -0.00316 | 0.002524 |
| 2001 | -0.00894 | -9.2E-05 | 0.01135 | -0.00764 | 0.007936 | 0.015829 |
| 2002 | 0.01062 | -0.0038 | -0.00191 | 0.005107 | -0.00212 | 0.008712 |
| 2003 | 0.042654 | -0.01028 | -0.0012 | 0.001957 | 0.005261 | 0.013487 |
| 2004 | 0.030411 | -0.01563 | 0.003192 | -0.00456 | 0.005391 | 0.013545 |
| 2005 | 0.014535 | 0.002757 | 0.001805 | -0.00403 | -0.00051 | 0.014087 |
| 2006 | 0.014779 | -0.00802 | -0.00497 | 0.001088 | -0.00388 | 0.007644 |
| 2007 | -0.00277 | -0.01078 | -0.00365 | 0.002658 | 0.001512 | 0.00297 |
| 2008 | 0.007646 | -0.02454 | 0.001028 | 0.003611 | -0.00474 | 0.00472 |
| 2009 | -0.04254 | 0.015383 | -0.00696 | 0.004433 | 0.000605 | 0.017581 |
| 2010 | 0.016327 | -0.01129 | -0.0078 | 0.011856 | 0.00503 | 0.010655 |
| 2011 | 0.002837 | -0.01489 | 0.002404 | 0.002116 | -0.00034 | 0.020201 |
| 2012 | -0.00854 | -0.00131 | 0.001988 | -0.00137 | 0.002523 | 0.013178 |
| 2013 | 0.035225 | -0.00144 | -0.00386 | 0.006164 | 0.004748 | 0.014668 |
| 2014 | 0.03462 | 0.005388 | 0.007941 | 0.006205 | 0.008211 | 0.013425 |
| 2015 | 0.043964 | -0.01669 | -0.00486 | 0.006156 | 0.007464 | 0.018428 |
| 2016 | 0.029422 | -0.0135 | 0.002313 | 0.002177 | 0.003807 | 0.006989 |
| 2017 | 0.014057 | -0.00624 | -0.00221 | -0.00502 | 0.00287 | 0.013038 |
| 2018 | -0.0217 | -0.00223 | -0.01152 | 0.015283 | 0.001547 | 0.012584 |
| 2019 | -0.00286 | -0.00303 | -0.00865 | 0.003681 | 0.002463 | 0.016534 |
| 2020 | 0.005098 | 0.014612 | -0.00512 | -0.00104 | -0.00111 | 0.01531 |

Notes: The annual average inflation-adjusted market returns, size, value, profitability, investment and momentum factors for the time-span of 27 years from Jan-1994 through Dec-2020 using average annual stocks returns.

Figure 1: Graphical presentation of Treasury Bills and Consumer Price Index (CPI):

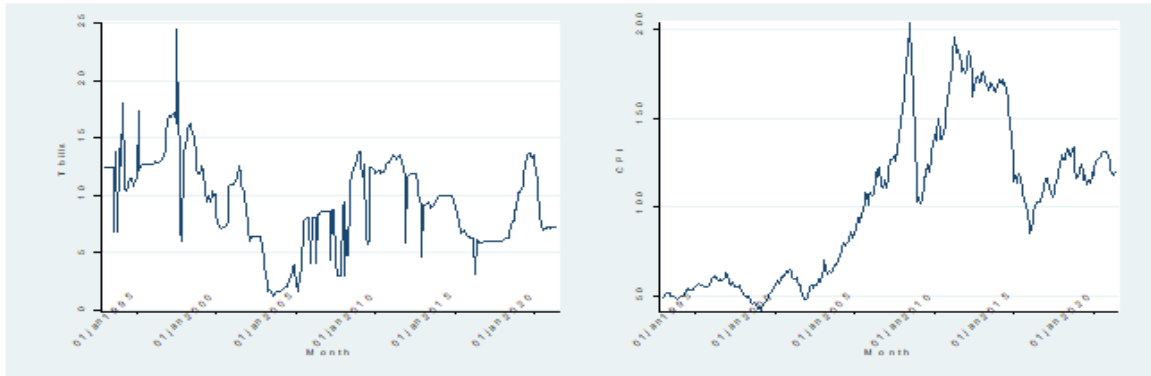


Figure 1 plots the graphical presentation of Government 3-months Treasury Bills and monthly Consumer Price Index (CPI) from Jan-1994 through Dec-2020. The graphics show how both influence the investors returns by representing the fluctuation of both the graphs.

Figure 2: Graphical presentation of RmInf, SMB, HML, RMW, CMA and WML:

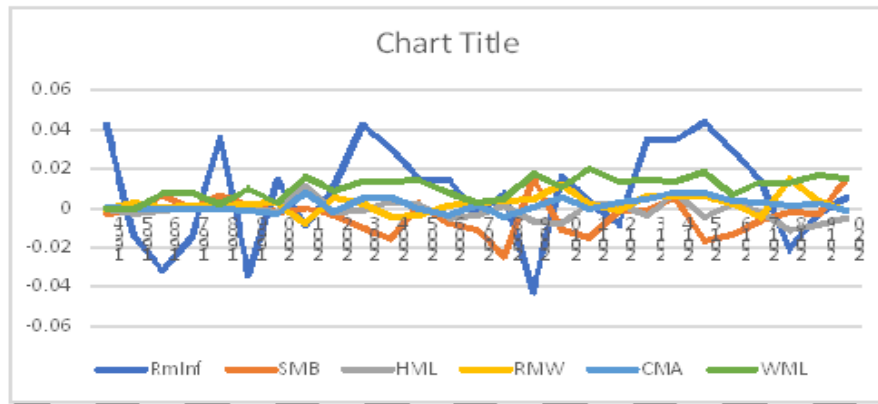


Figure 1 depicts the Govt. 3-months Treasury Bills Rate and monthly Consumer Price index for 1994-2020. On the other hand, Figure 2 plots the annual average returns for inflation-adjusted market, size, value, profitability, investment and momentum factors for time-span 1994-2022. The annual average inflation-adjusted market returns illustrate more fluctuations as the graphical trend depicts volatility for the whole time-span. The size pattern shows high volatility in 2008-2009 which demonstrates that investors switched their investment to small firms during financial crises to save their investment. The profitability patten shows slight fluctuations in 2017-2018 while other factors exhibit almost normal behaviour for the sample period.

Empirical Results

In this study, we empirically analyze the nexus of various factors with excess inflation-adjusted portfolio returns. Based on the methodology described above, the study first investigated traditional CAPM model while employing time-series simple OLS regression using inflation adjusted 25 equal-weighted excess returns on LHS as dependent variables and inflation adjusted excess market returns on RHS as independent variable.

Factor Spanning / Redundancy Tests

As Fama and French (2018) comment on the significance of RHS tests (factor spanning test) and argue that it justifies the contribution of additional factor to be included in the analysis. This approach is proposed by (Huberman & Kandel, 1987) as the mean-variance

spanning which infers whether one factor can be explained with the support of other factors. It examines the viability of combination of other factors to access whether factor is redundant or not. In case, one factor is

captured by remaining factors, it is useless to be the part of the model (Fama & French, 2016).

Table 6: Factor Spanning Tests

| VARIABLES | (1) RmInf | (2) SMB | (3) HML | (4) RMW | (5) CMA | (6) WML |
|-----------|--------------|------------|------------|------------|------------|------------|
| RmInf | . | -0.010 | 0.001 | 0.004 | 0.003 | -0.024 |
| | (.) | (-0.483) | (0.048) | (0.379) | (0.340) | (-1.524) |
| SMB | -0.071 | . | 0.051* | -0.007 | -0.007 | 0.185*** |
| | (-0.483) | (.) | (1.794) | (-0.280) | (-0.314) | (4.464) |
| HML | 0.014 | 0.186* | . | -0.263*** | 0.306*** | -0.517*** |
| | (0.048) | (1.794) | (.) | (-5.498) | (7.406) | (-6.770) |
| RMW | 0.116 | -0.032 | -0.314*** | . | 0.173*** | -0.057 |
| | (0.379) | (-0.280) | (-5.498) | (.) | (3.615) | (-0.640) |
| CMA | 0.117 | -0.040 | 0.458*** | 0.217*** | . | 0.636*** |
| | (0.340) | (-0.314) | (7.406) | (3.615) | (.) | (6.812) |
| WML | -0.286 | 0.302*** | -0.232*** | -0.021 | 0.191*** | . |
| | (-1.524) | (4.464) | (-6.770) | (-0.640) | (6.812) | (.) |
| Constant | 0.010* | -0.006*** | 0.001 | 0.002* | -0.000 | 0.010*** |
| | (1.890) | (-3.256) | (0.866) | (1.855) | (-0.351) | (7.009) |
| R-squared | 0.010 | 0.065 | 0.247 | 0.101 | 0.211 | 0.233 |

t-statistics in parentheses, *** p<0.01, ** p<0.05, * p<0.1

Notes: Table 6 demonstrates the factor spanning tests using all independent variables (factors). Each factor is used as dependent variable while other factors as independent variables for testing redundancy test.

As Fama and French (2018) comment on the significance of RHS tests (factor spanning test) and argue that it justifies the contribution of additional factor to be included in the analysis. This approach is proposed by (Huberman & Kandel, 1987) as the mean-variance spanning which infers whether one factor can be explained with the support of other factors. It examines the viability of combination of other factors to access

whether factor is redundant or not. In case, one factor is captured by remaining factors, it is useless to be the part of the model (Fama & French, 2016).

Capital Asset Pricing Model (CAPM) Regression Results

Table 7: Results & performance of traditional CAPM using inflation adjusted value-weighted portfolios

| Alpha | L_B M | 2 | 3 | 4 | H_B M | T-value | L_B M | 2 | 3 | 4 | H_B M |
|--------------|-----------|-----------|-----------|-----------|-----------|--------------|--------|--------|--------|--------|--------|
| Small | -0.020*** | -0.019*** | -0.018*** | -0.019*** | -0.023*** | Small | -5.774 | -4.465 | -4.766 | -4.181 | -6.336 |
| 2 | -0.017*** | -0.017*** | -0.012*** | -0.017*** | -0.022*** | 2 | -4.007 | -4.503 | -3.042 | -4.209 | -5.551 |
| 3 | -0.019*** | -0.014*** | -0.014*** | -0.016*** | -0.013*** | 3 | -4.923 | -3.579 | -4.009 | -4.380 | -3.738 |
| 4 | -0.018*** | -0.016*** | -0.016*** | -0.014*** | -0.013*** | 4 | -5.249 | -4.652 | -4.704 | -3.558 | -3.587 |
| Big | -0.018*** | -0.016*** | -0.014*** | -0.012*** | -0.012*** | Big | -4.245 | -4.548 | -4.217 | -3.229 | -3.462 |
| RmInf | L_B M | 2 | 3 | 4 | H_B M | T-value | L_B M | 2 | 3 | 4 | H_B M |
| Small | 0.153*** | 0.116** | 0.113*** | 0.147*** | 0.118*** | Small | 3.811 | 2.447 | 2.652 | 2.858 | 2.814 |
| 2 | 0.110** | 0.159*** | 0.066 | 0.124*** | 0.157*** | 2 | 2.258 | 3.744 | 1.455 | 2.727 | 3.507 |
| 3 | 0.132*** | 0.144*** | 0.133*** | 0.135*** | 0.127*** | 3 | 2.982 | 3.353 | 3.454 | 3.322 | 3.266 |
| 4 | 0.154*** | 0.140*** | 0.127*** | 0.123*** | 0.109*** | 4 | 3.866 | 3.599 | 3.328 | 2.853 | 2.607 |
| Big | 0.164*** | 0.141*** | 0.165*** | 0.159*** | 0.163*** | Big | 3.441 | 3.510 | 4.532 | 3.716 | 4.083 |
| Adj. R2 | L_B M | 2 | 3 | 4 | H_B M | F-value | L_B M | 2 | 3 | 4 | H_B M |
| Small | 0.043 | 0.018 | 0.021 | 0.025 | 0.024 | Small | 14.45 | 5.91 | 6.98 | 8.12 | 7.88 |
| 2 | 0.016 | 0.042 | 0.007 | 0.023 | 0.037 | 2 | 5.06 | 13.93 | 2.06 | 7.38 | 12.43 |
| 3 | 0.027 | 0.034 | 0.036 | 0.033 | 0.032 | 3 | 8.83 | 11.15 | 11.82 | 10.95 | 10.57 |
| 4 | 0.044 | 0.039 | 0.033 | 0.025 | 0.021 | 4 | 14.97 | 12.86 | 10.98 | 8.28 | 6.73 |
| Big | 0.035 | 0.037 | 0.060 | 0.041 | 0.049 | Big | 11.94 | 12.20 | 20.42 | 13.65 | 16.51 |

Notes: Table 7 depicts the findings of CAPM using inflation adjusted value-weighted portfolios including intercept, coefficient and adjusted R-square results.

Table 7 demonstrates the results of CAPM which shows that 22 out of 25 (22/25) portfolios present statistically strongly significant coefficient results at the probability of 0.001% similar to (Azam & Naveed, 2022). The 2 portfolios (4L and B2) show statistically moderately significant results at the probability of 0.01% while only one portfolio (43) which posits statistically insignificant coefficient result. The results conclude that inflation adjusted market excess returns explains the inflation adjusted excess portfolio returns using financial and non-financial firms enlisted on PSX for the time period 1994-2022.

Table 7 indicates the findings of inflation adjusted CAPM which are extracted from employing OLS simple-regression estimation. The findings demonstrate highly statistically significant and positive results as all market inflation adjusted factor coefficients show significant values except one portfolios (23, $\beta = 0.066$, t-value =

1.455). The findings support the theory and establish significant and positive nexus with inflation adjusted real excess portfolio returns. The adjusted R-squares range from 0.007 to 0.049 for portfolio 23 and BH respectively. The overall results of the model based on F-statistics show significant findings except portfolio 23 ($F = 2.06$). Fama and French (2015) calculate the average absolute alpha (intercept) coefficient (henceforth AAAC) for evaluating the model's validity. The AAAC of CAPM is -0.01636 (1.64 percent), indicating that the monthly yields have a 1.64 percent pricing errors, according to (Lohano & Kashif, 2018). These inverse nexus identify overvalued portfolios and determines the invalidity of CAPM (Lohano & Kashif, 2018).

Fama and French (1993) three-factor model Regression Results

Table 8: Results & performance of inflation-adjusted FF3FM using value-weighted portfolios

| Alpha | H-B/M | 4 | 3 | 2 | L-B/M | T-Value | H-B/M | 4 | 3 | 2 | L-B/M |
|--------------|-----------|-----------|-----------|-----------|-----------|--------------|--------|--------|--------|--------|--------|
| Small | -0.020*** | -0.017*** | -0.015*** | -0.014*** | -0.014*** | Small | -5.339 | -5.139 | -4.842 | -3.791 | -4.212 |
| 2 | -0.018*** | -0.016*** | -0.017*** | -0.015*** | -0.015*** | 2 | -5.341 | -4.734 | -5.183 | -3.965 | -4.100 |
| 3 | -0.018*** | -0.012*** | -0.013*** | -0.017*** | -0.013*** | 3 | -4.713 | -3.268 | -3.867 | -4.579 | -3.638 |
| 4 | -0.013*** | -0.015*** | -0.010** | -0.016*** | -0.021*** | 4 | -3.367 | -4.044 | -2.551 | -3.978 | -5.372 |
| Big | -0.018*** | -0.014*** | -0.016*** | -0.016*** | -0.022*** | Big | -5.282 | -3.852 | -4.380 | -3.717 | -6.022 |
| RmInf | H-B/M | 4 | 3 | 2 | L-B/M | T-Value | H-B/M | 4 | 3 | 2 | L-B/M |
| Small | 0.142*** | 0.125*** | 0.152*** | 0.150*** | 0.151*** | Small | 3.378 | 3.366 | 4.431 | 3.598 | 3.965 |
| 2 | 0.146*** | 0.130*** | 0.118*** | 0.122*** | 0.101** | 2 | 3.761 | 3.490 | 3.159 | 2.840 | 2.462 |
| 3 | 0.126*** | 0.144*** | 0.133*** | 0.133*** | 0.130*** | 3 | 2.941 | 3.366 | 3.417 | 3.251 | 3.326 |
| 4 | 0.116*** | 0.164*** | 0.074* | 0.129*** | 0.163*** | 4 | 2.605 | 3.926 | 1.683 | 2.859 | 3.662 |
| Big | 0.158*** | 0.122*** | 0.121*** | 0.164*** | 0.129*** | Big | 4.125 | 2.922 | 2.885 | 3.416 | 3.160 |
| SMB | H-B/M | 4 | 3 | 2 | L-B/M | T-Value | H-B/M | 4 | 3 | 2 | L-B/M |
| Small | -0.966*** | -0.598*** | -0.558*** | -0.479*** | -0.605*** | Small | -8.669 | -6.066 | -6.123 | -4.329 | -5.981 |
| 2 | -0.262** | -0.275*** | -0.441*** | -0.249** | -0.446*** | 2 | -2.539 | -2.783 | -4.453 | -2.184 | -4.081 |
| 3 | -0.034 | 0.132 | 0.021 | -0.180* | 0.120 | 3 | -0.298 | 1.162 | 0.206 | -1.659 | 1.156 |
| 4 | 0.650*** | 0.362*** | 0.525*** | 0.284** | 0.259** | 4 | 5.491 | 3.269 | 4.512 | 2.365 | 2.190 |
| Big | 0.453*** | 0.703*** | 0.439*** | 0.907*** | 0.532*** | Big | 4.444 | 6.355 | 3.946 | 7.107 | 4.902 |
| HML | H-B/M | 4 | 3 | 2 | L-B/M | T-Value | H-B/M | 4 | 3 | 2 | L-B/M |
| Small | 1.017*** | 0.861*** | 0.532*** | -0.194 | -0.010 | Small | 5.351 | 5.122 | 3.429 | -1.027 | -0.057 |
| 2 | 0.722*** | 0.868*** | 0.126 | -0.386** | -0.139 | 2 | 4.100 | 5.145 | 0.750 | -1.990 | -0.745 |
| 3 | 0.971*** | 0.495** | 0.156 | -0.175 | -0.167 | 3 | 4.998 | 2.555 | 0.888 | -0.949 | -0.945 |
| 4 | 1.120*** | 0.484** | 0.179 | -0.108 | -0.129 | 4 | 5.552 | 2.567 | 0.900 | -0.526 | -0.639 |
| Big | 0.616*** | 1.356*** | 0.068 | -0.039 | -0.119 | Big | 3.545 | 7.187 | 0.360 | -0.177 | -0.644 |
| Adj-R2 | H-B/M | 4 | 3 | 2 | L-B/M | F-Value | H-B/M | 4 | 3 | 2 | L-B/M |
| Small | 0.2572 | 0.1812 | 0.1731 | 0.0899 | 0.1372 | Small | 38.28 | 24.82 | 23.54 | 11.63 | 18.12 |
| 2 | 0.0983 | 0.1200 | 0.0819 | 0.0436 | 0.0630 | 2 | 12.74 | 15.68 | 10.60 | 5.91 | 8.24 |
| 3 | 0.0891 | 0.0492 | 0.0292 | 0.0356 | 0.0294 | 3 | 11.53 | 6.57 | 4.24 | 4.97 | 4.26 |
| 4 | 0.1735 | 0.0851 | 0.0610 | 0.0309 | 0.0433 | 4 | 23.61 | 11.02 | 8.00 | 4.43 | 5.87 |
| Big | 0.1276 | 0.2408 | 0.0591 | 0.1503 | 0.0843 | Big | 16.75 | 35.14 | 7.77 | 20.04 | 10.91 |

Table 8 demonstrates the FF3FM results which are extracted from employing OLS multiple-regression estimations. The inflation adjusted performance of value-weighted real returns is used to justify the inflation-adjusted market returns, size and value risk patterns in PSX. The t-values are shown on right hand-side.

Table 8 predicts the findings of inflation adjusted excess portfolio regressed on market, size and value risk premiums using FF3FM which demonstrate highly statistically significant but positive results for 23/25 mimicking portfolios while 2 portfolios (2L and 43) show moderately and weakly respectively but significant nexus with portfolio returns having t-value greater than 2. On the other hand, size factor shows mixed results as 12/25 portfolios show negative while 13/25 show positive nexus with inflation adjusted excess portfolio returns.

Table 8 exhibits time-series OLS multiple regression results extracted from FF3FM using the Inflation adjusted real-excess portfolio monthly returns data. The outcomes demonstrate supportive results to the theory as all three-factors show statistically significant nexus with real-excess portfolio returns. The real market excess returns show highly significant and positive relationship with inflation adjusted real excess portfolio returns (IAREPR). The corresponding t-statistics also display statistically acceptable range values for all 25 portfolios. Similarly, size-pattern (SMB) displays statistically significant

relationship with excess returns as all small market-cap stocks portfolios show positive and highly statistically significant values while big market-cap stocks portfolios exhibit statistically significant but negative nexus with inflation-adjusted excess portfolios returns which designates the existence of the size-pattern in the market. Moreover, value-pattern (HML) also shows statistically highly significant results in line with the theory that high B|M ratio stocks portfolios (HBMSP) outperform low B|M ratio (LBMSP) stocks portfolios. The findings confirm the theory as HBMSP demonstrate statistically highly significant and positive nexus while LBMSP show almost insignificant and negative nexus with IAREPR which confirms the existence of value factor in the

market. Moreover, the t-statistics of all HBMSP show greater than 2 which also confirm the existence of value-pattern in the market. The overall results of the model reveal that FF3FM is appropriate model for the PSX to explain average inflation adjusted excess portfolio returns. Moreover, the alpha values of all 25 portfolios shows statistically significant but inverse nexus. The AAAC of FF3FM is -0.0158 (1.58 percent), indicating that the monthly yields have a 1.58 percent pricing errors, according to (Lohano & Kashif, 2018). These inverse nexus identify overvalued portfolios and determines the invalidity of FF3FM (Lohano & Kashif, 2018).

Carhart (1997) four-factor model Regression Results

Table 9: Results & performance of inflation-adjusted C4FM using value-weighted portfolios

| Alpha | H-B M | 4 | 3 | 2 | L-B M | T-value | H-B M | 4 | 3 | 2 | L-B M |
|---------------------|-----------|-----------|-----------|-----------|-----------|--------------|--------|--------|--------|--------|--------|
| Small | -0.013*** | -0.012*** | -0.010*** | -0.012*** | -0.013*** | Small | -3.387 | -3.581 | -3.252 | -3.056 | -3.501 |
| 2 | -0.015*** | -0.014*** | -0.017*** | -0.016*** | -0.015*** | 2 | -4.125 | -4.080 | -4.752 | -3.820 | -3.826 |
| 3 | -0.013*** | -0.014*** | -0.016*** | -0.017*** | -0.010** | 3 | -3.177 | -3.335 | -4.261 | -4.349 | -2.570 |
| 4 | -0.012*** | -0.015*** | -0.010** | -0.014*** | -0.014*** | 4 | -2.894 | -3.852 | -2.373 | -3.154 | -3.421 |
| Big | -0.015*** | -0.012*** | -0.019*** | -0.010** | -0.017*** | Big | -4.053 | -3.045 | -4.830 | -2.266 | -4.388 |
| RmInf | H-B M | 4 | 3 | 2 | L-B M | T-value | H-B M | 4 | 3 | 2 | L-B M |
| Small | 0.126*** | 0.115*** | 0.143*** | 0.146*** | 0.148*** | Small | 3.093 | 3.157 | 4.230 | 3.509 | 3.885 |
| 2 | 0.139*** | 0.128*** | 0.117*** | 0.122*** | 0.102** | 2 | 3.587 | 3.421 | 3.143 | 2.840 | 2.467 |
| 3 | 0.115*** | 0.147*** | 0.138*** | 0.134*** | 0.123*** | 3 | 2.719 | 3.428 | 3.568 | 3.267 | 3.173 |
| 4 | 0.114** | 0.165*** | 0.074* | 0.124*** | 0.147*** | 4 | 2.551 | 3.937 | 1.691 | 2.747 | 3.382 |
| Big | 0.151*** | 0.117*** | 0.128*** | 0.152*** | 0.118*** | Big | 3.960 | 2.814 | 3.053 | 3.202 | 2.933 |
| SMB | H-B M | 4 | 3 | 2 | L-B M | T-value | H-B M | 4 | 3 | 2 | L-B M |
| Small | -0.839*** | -0.511*** | -0.474*** | -0.441*** | -0.574*** | Small | -7.523 | -5.112 | -5.147 | -3.870 | -5.509 |
| 2 | -0.201* | -0.253** | -0.435*** | -0.262** | -0.446*** | 2 | -1.899 | -2.475 | -4.261 | -2.230 | -3.953 |
| 3 | 0.066 | 0.109 | -0.024 | -0.186* | 0.181* | 3 | 0.573 | 0.933 | -0.229 | -1.662 | 1.699 |
| 4 | 0.669*** | 0.355*** | 0.527*** | 0.331*** | 0.394*** | 4 | 5.482 | 3.105 | 4.382 | 2.676 | 3.326 |
| Big | 0.516*** | 0.746*** | 0.383*** | 1.015*** | 0.629*** | Big | 4.947 | 6.551 | 3.354 | 7.842 | 5.726 |
| HML | H-B M | 4 | 3 | 2 | L-B M | T-value | H-B M | 4 | 3 | 2 | L-B M |
| Small | 0.782*** | 0.702*** | 0.380** | -0.263 | -0.066 | Small | 4.094 | 4.099 | 2.407 | -1.345 | -0.367 |
| 2 | 0.608*** | 0.826*** | 0.117 | -0.363* | -0.138 | 2 | 3.360 | 4.727 | 0.669 | -1.802 | -0.715 |
| 3 | 0.788*** | 0.537*** | 0.240 | -0.163 | -0.278 | 3 | 3.982 | 2.673 | 1.321 | -0.854 | -1.526 |
| 4 | 1.084*** | 0.497** | 0.177 | -0.193 | -0.377* | 4 | 5.183 | 2.540 | 0.858 | -0.912 | -1.860 |
| Big | 0.500*** | 1.278*** | 0.171 | -0.237 | -0.297 | Big | 2.802 | 6.553 | 0.874 | -1.070 | -1.579 |
| WML | H-B M | 4 | 3 | 2 | L-B M | T-value | H-B M | 4 | 3 | 2 | L-B M |
| Small | -0.609*** | -0.415*** | -0.396*** | -0.179 | -0.145 | Small | -4.611 | -3.502 | -3.624 | -1.329 | -1.177 |
| 2 | -0.293** | -0.108 | -0.025 | 0.062 | 0.001 | 2 | -2.342 | -0.894 | -0.204 | 0.444 | 0.008 |
| 3 | -0.476*** | 0.108 | 0.217* | 0.030 | -0.288** | 3 | -3.478 | 0.777 | 1.727 | 0.227 | -2.284 |
| 4 | -0.093 | 0.033 | -0.006 | -0.221 | -0.644*** | 4 | -0.643 | 0.245 | -0.041 | -1.513 | -4.587 |
| Big | -0.299** | -0.202 | 0.266** | -0.516*** | -0.462*** | Big | -2.424 | -1.498 | 1.968 | -3.364 | -3.553 |
| Adj. R ² | H-B M | 4 | 3 | 2 | L-B M | F-value | H-B M | 4 | 3 | 2 | L-B M |
| Small | 0.310 | 0.219 | 0.213 | 0.103 | 0.149 | Small | 35.83 | 22.33 | 21.60 | 9.18 | 13.95 |
| 2 | 0.122 | 0.130 | 0.091 | 0.053 | 0.072 | 2 | 11.05 | 11.95 | 7.94 | 4.47 | 6.16 |
| 3 | 0.131 | 0.060 | 0.047 | 0.045 | 0.054 | 3 | 11.97 | 5.07 | 3.95 | 3.73 | 4.54 |
| 4 | 0.182 | 0.094 | 0.070 | 0.047 | 0.110 | 4 | 17.78 | 8.25 | 5.98 | 3.91 | 9.93 |
| Big | 0.151 | 0.253 | 0.079 | 0.187 | 0.127 | Big | 14.22 | 27.02 | 6.85 | 18.34 | 11.63 |

Notes: Table 9 depicts the findings of C4FM using inflation adjusted value-weighted portfolios including intercept, coefficient and adjusted R-square results.

Table 9 summarizes the Carhart (1997) four-factor model findings. The inflation-adjusted market pattern demonstrates statistically significant for all portfolios

showing inconsistency with (Azam & Naveed, 2022). The t-values also indicate significance for the associated coefficients. The size pattern also exhibits statistically significant results and supports the theory which indicates

that portfolios having small market-caps outperform portfolios having big market-caps. The all-small market-cap firms' portfolios demonstrate highly statistically significant and positive nexus with inflation adjusted excess portfolio returns (IAEPR). Conversely, all big market-cap firms' portfolios exhibit statistically significant but inverse relationship with IAEPR which support the theory of size factor. The value pattern also supports the theory as value-portfolios (higher B|M ratio) demonstrate statistically significant and positive coefficients while growth-portfolios (lower B|M ratio) show inverse but statistically insignificant results except two portfolios (42 and 2L having $\beta = -0.363$ and -0.377 respectively). The momentum pattern shows mix and tenuous results as 12 out of 25 show significant coefficients but inverse nexus with IAEPR. The adjusted R-square values indicate more influential impact on IAEPR for high B|M ratio and small market-cap portfolios. Moreover, the alpha values of all 25 portfolios shows statistically significant but inverse nexus similar to CAPM and FF3FM. The AAAC of C4FM is -0.0138 (1.38 percent), indicating that the monthly yields have a 1.38 percent pricing errors, according to (Lohano & Kashif, 2018). These inverse nexus identify overvalued portfolios and determines the invalidity of C4FM (Lohano & Kashif, 2018).

Fama & French (2015) five-factor model Regression Results

Table 10 summarizes the estimation results of FF5FM using OLS regression technique. Similar to FF3FM and C4FM, the inflation-adjusted market demonstrates highly statistically significant coefficients for all portfolios. The t-value ranges from 1.676 to 4.498. The size and value-patterns demonstrate almost similar and statistically significant results as C4FM supporting with theories. Moreover, profitability pattern shows 16 out of 25 coefficients statistically significant impact on IAEPR. Conversely, the investment pattern confirms statistically insignificant findings except three portfolios (34, 33 and 42 with $\beta = 0.526, 0.667$ and 0.435 respectively). The F-values ranges from 3.15 to 25.07 which indicate declining situation as compare to FF3FM and C4FM findings. Moreover, the alpha values of all 25 portfolios shows statistically significant but inverse nexus similar to CAPM, FF3FM and C4FM. The AAAC of FF5FM is -0.0154 (1.54 percent), indicating that the monthly yields have a 1.54 percent pricing errors, according to (Lohano & Kashif, 2018). These inverse nexus identify overvalued portfolios and determines the invalidity of FF5FM (Lohano & Kashif, 2018).

Table 10: Estimated results of Fama & French (2015) five-factor model

| Alpha | H-B M | 4 | 3 | 2 | L-B M | T-value | H-B M | 4 | 3 | 2 | L-B M |
|--------------|-----------|-----------|-----------|-----------|-----------|--------------|--------|--------|--------|--------|--------|
| Small | -0.018*** | -0.016*** | -0.014*** | -0.015*** | -0.014*** | Small | -4.897 | -4.817 | -4.566 | -3.980 | -3.984 |
| 2 | -0.018*** | -0.015*** | -0.017*** | -0.015*** | -0.014*** | 2 | -5.132 | -4.590 | -5.162 | -3.907 | -3.944 |
| 3 | -0.017*** | -0.013*** | -0.014*** | -0.017*** | -0.012*** | 3 | -4.517 | -3.349 | -4.030 | -4.731 | -3.357 |
| 4 | -0.012*** | -0.016*** | -0.010** | -0.015*** | -0.020*** | 4 | -3.086 | -4.191 | -2.591 | -3.779 | -5.030 |
| Big | -0.019*** | -0.013*** | -0.015*** | -0.016*** | -0.020*** | Big | -5.471 | -3.620 | -4.085 | -3.674 | -5.703 |
| RmInf | H-B M | 4 | 3 | 2 | L-B M | T-value | H-B M | 4 | 3 | 2 | L-B M |
| Small | 0.143*** | 0.127*** | 0.154*** | 0.150*** | 0.154*** | Small | 3.445 | 3.457 | 4.498 | 3.608 | 4.055 |
| 2 | 0.147*** | 0.133*** | 0.120*** | 0.123*** | 0.105** | 2 | 3.785 | 3.610 | 3.228 | 2.879 | 2.589 |
| 3 | 0.129*** | 0.146*** | 0.135*** | 0.133*** | 0.131*** | 3 | 3.031 | 3.436 | 3.528 | 3.259 | 3.371 |
| 4 | 0.119*** | 0.164*** | 0.073* | 0.132*** | 0.163*** | 4 | 2.689 | 3.938 | 1.676 | 2.933 | 3.666 |
| Big | 0.158*** | 0.125*** | 0.124*** | 0.166*** | 0.133*** | Big | 4.129 | 3.041 | 3.011 | 3.486 | 3.374 |
| SMB | H-B M | 4 | 3 | 2 | L-B M | T-value | H-B M | 4 | 3 | 2 | L-B M |
| Small | -0.963*** | -0.600*** | -0.559*** | -0.486*** | -0.611*** | Small | -8.712 | -6.110 | -6.147 | -4.386 | -6.056 |
| 2 | -0.266** | -0.290*** | -0.454*** | -0.266** | -0.463*** | 2 | -2.568 | -2.958 | -4.602 | -2.350 | -4.293 |
| 3 | -0.046 | 0.113 | -0.003 | -0.188* | 0.125 | 3 | -0.402 | 0.998 | -0.027 | -1.734 | 1.202 |
| 4 | 0.643*** | 0.351*** | 0.532*** | 0.275** | 0.267** | 4 | 5.467 | 3.169 | 4.556 | 2.294 | 2.253 |
| Big | 0.443*** | 0.691*** | 0.429*** | 0.889*** | 0.517*** | Big | 4.350 | 6.311 | 3.902 | 6.992 | 4.915 |
| HML | H-B M | 4 | 3 | 2 | L-B M | T-value | H-B M | 4 | 3 | 2 | L-B M |
| Small | 0.952*** | 0.778*** | 0.469*** | -0.219 | -0.121 | Small | 4.697 | 4.321 | 2.813 | -1.079 | -0.652 |
| 2 | 0.656*** | 0.679*** | -0.018 | -0.572*** | -0.366* | 2 | 3.457 | 3.782 | -0.101 | -2.751 | -1.850 |
| 3 | 0.801*** | 0.313 | -0.068 | -0.221 | -0.178 | 3 | 3.855 | 1.509 | -0.365 | -1.112 | -0.935 |
| 4 | 0.974*** | 0.415** | 0.258 | -0.257 | -0.113 | 4 | 4.516 | 2.044 | 1.206 | -1.173 | -0.520 |
| Big | 0.568*** | 1.163*** | -0.122 | -0.239 | -0.378* | Big | 3.036 | 5.792 | -0.604 | -1.025 | -1.960 |
| RMW | H-B M | 4 | 3 | 2 | L-B M | T-value | H-B M | 4 | 3 | 2 | L-B M |
| Small | -0.567** | -0.488** | -0.359* | 0.140 | -0.476** | Small | -2.415 | -2.341 | -1.860 | 0.596 | -2.221 |
| 2 | -0.295 | -0.606*** | -0.363* | -0.469* | -0.746*** | 2 | -1.344 | -2.912 | -1.731 | -1.948 | -3.258 |
| 3 | -0.614** | -0.364 | -0.433** | 0.067 | -0.268 | 3 | -2.552 | -1.516 | -1.999 | 0.291 | -1.217 |
| 4 | -0.651*** | 0.008 | 0.248 | -0.570** | -0.241 | 4 | -2.609 | 0.035 | 1.000 | -2.241 | -0.957 |
| Big | 0.093 | -0.723*** | -0.785*** | -0.528* | -1.068*** | Big | 0.430 | -3.110 | -3.368 | -1.955 | -4.781 |

| CMA | H-B M | 4 | 3 | 2 | L-B M | T-value | H-B M | 4 | 3 | 2 | L-B M |
|---------------------|--------|---------|----------|--------|--------|--------------|--------|--------|--------|-------|--------|
| Small | -0.264 | -0.084 | -0.054 | 0.272 | 0.062 | Small | -1.070 | -0.386 | -0.268 | 1.097 | 0.277 |
| 2 | 0.023 | 0.311 | 0.345 | 0.435* | 0.360 | 2 | 0.098 | 1.421 | 1.566 | 1.720 | 1.494 |
| 3 | 0.213 | 0.526** | 0.667*** | 0.295 | -0.220 | 3 | 0.840 | 2.081 | 2.931 | 1.220 | -0.950 |
| 4 | 0.056 | 0.349 | -0.139 | 0.158 | -0.328 | 4 | 0.212 | 1.411 | -0.534 | 0.591 | -1.239 |
| Big | 0.333 | 0.214 | 0.137 | 0.450 | 0.189 | Big | 1.463 | 0.875 | 0.557 | 1.586 | 0.802 |
| Adj. R ² | H-B M | 4 | 3 | 2 | L-B M | F-value | H-B M | 4 | 3 | 2 | L-B M |
| Small | 0.283 | 0.204 | 0.191 | 0.104 | 0.158 | Small | 25.07 | 16.32 | 14.98 | 7.36 | 11.96 |
| 2 | 0.112 | 0.153 | 0.103 | 0.069 | 0.104 | 2 | 8.00 | 11.47 | 7.33 | 4.72 | 7.37 |
| 3 | 0.116 | 0.074 | 0.070 | 0.050 | 0.047 | 3 | 8.35 | 5.12 | 4.77 | 3.34 | 3.15 |
| 4 | 0.199 | 0.099 | 0.073 | 0.055 | 0.061 | 4 | 15.76 | 7.03 | 5.01 | 3.69 | 4.13 |
| Big | 0.143 | 0.270 | 0.100 | 0.172 | 0.154 | Big | 10.61 | 23.55 | 7.06 | 13.22 | 11.54 |

Notes: Table 10 depicts the findings of FF5FM using inflation adjusted value-weighted portfolios including intercept, coefficient and adjusted R-square results. The right side shows T-values and F-values for 25 portfolios.

Momentum augmented Fama & French (2015) five-factor model Regression Results

Table 11 shows momentum augmented FF5FM using time-series OLS regression approach. The findings show that market-factor demonstrates statistically significant nexus with average inflation adjusted excess portfolio returns showing inconsistency with (Azam & Naveed, 2022). Only one portfolio shows weak significant results (portfolio 23, $\beta = 0.074$ with t-states = 1.678). The t-values except portfolio 23, ranges from 2.542 to 4.266 which indicates that market pattern contributes statistically significant nexus with portfolio returns in the market. Similarly, the size-pattern demonstrates statistically significant and positive nexus with IAEPR for small market-cap stocks portfolios while the big market-cap stocks portfolios show significant but negative relationship with IAEPR which indicates that size-factor exists in the market. Likewise, value-pattern shows statistically significant nexus with IAEPR as portfolios having high B|M ratios except one portfolio (34, having $\beta = 0.321$, with t-states = 1.449) all portfolios show positive significant values. On the contrary, portfolios having low B|M ratios exhibit negative but significant results except portfolios (BL, 32 and 3L having t-states = -1.056, -0.238 and -0.321 respectively) which designates the value-pattern significant contribution in the market. The profitability pattern determines mix and tenuous results in terms of magnitude as except five portfolios

(SH, 24, 23, B2 and 32), all portfolios show negative relationship with IAEPR. Moreover, 16 out of 25 portfolios show statistically significant results while only 9 portfolios show insignificant results. On the contrary, investment pattern displays poor results in explaining the IAEPR in the market. In addition, the sixth factor is momentum which demonstrates better performance as the results show 14 significant estimated coefficients. The high B|M stocks portfolios show positively significant results for 6 out of 10 portfolios while the low B|M stocks portfolios show significant results for 6 out of 10 portfolios but show negative magnitudes which confirms the value stocks momentum excess portfolio returns. In addition, the adjusted R-square also significant growth as compare to previous explained models. Furthermore, the evidence of F-values also indicate that momentum augmented FF5FM produces better results as compare to other baseline models presented above in the study. Based on F-value, the overall portfolios show statistically significant results except three portfolios (42, 22 and 3L with F-value = 3.92, 3.61 and 3.33) respectively. Moreover, the alpha values of all 25 portfolios shows statistically significant but inverse nexus similar to CAPM, FF3FM, C4FM and FF5FM. The AAAC of MFF5FM is -0.01312 (1.31 percent), indicating that the monthly yields have a 1.31 percent pricing errors, according to (Lohano & Kashif, 2018). These inverse nexus identify overvalued portfolios and determines the invalidity of MFF5FM (Lohano & Kashif, 2018).

Table 11: Estimates for Momentum adjusted FF5FM

| Alpha | H-B M | 4 | 3 | 2 | L-B M | T-value | H-B M | 4 | 3 | 2 | L-B M |
|--------------|-----------|-----------|-----------|-----------|-----------|--------------|--------|--------|--------|--------|--------|
| Small | -0.012*** | -0.012*** | -0.010*** | -0.012*** | -0.012*** | Small | -3.142 | -3.341 | -3.045 | -3.076 | -3.279 |
| 2 | -0.015*** | -0.013*** | -0.016*** | -0.015*** | -0.014*** | 2 | -3.960 | -3.809 | -4.562 | -3.615 | -3.535 |
| 3 | -0.012*** | -0.013*** | -0.015*** | -0.017*** | -0.009** | 3 | -2.914 | -3.165 | -4.062 | -4.335 | -2.439 |
| 4 | -0.011*** | -0.015*** | -0.010** | -0.013*** | -0.014*** | 4 | -2.650 | -3.816 | -2.460 | -2.924 | -3.291 |
| Big | -0.015*** | -0.011*** | -0.018*** | -0.009** | -0.015*** | Big | -4.060 | -2.752 | -4.571 | -2.045 | -4.030 |
| RmInf | H-B M | 4 | 3 | 2 | L-B M | T-value | H-B M | 4 | 3 | 2 | L-B M |
| Small | 0.128*** | 0.117*** | 0.143*** | 0.144*** | 0.150*** | Small | 3.166 | 3.216 | 4.266 | 3.456 | 3.940 |
| 2 | 0.139*** | 0.129*** | 0.117*** | 0.122*** | 0.103** | 2 | 3.598 | 3.488 | 3.153 | 2.857 | 2.542 |
| 3 | 0.115*** | 0.146*** | 0.138*** | 0.132*** | 0.125*** | 3 | 2.762 | 3.427 | 3.583 | 3.224 | 3.208 |
| 4 | 0.116*** | 0.163*** | 0.074* | 0.125*** | 0.147*** | 4 | 2.623 | 3.896 | 1.678 | 2.788 | 3.395 |
| Big | 0.148*** | 0.119*** | 0.131*** | 0.150*** | 0.120*** | Big | 3.911 | 2.891 | 3.187 | 3.218 | 3.109 |

| SMB | H-B M | 4 | 3 | 2 | L-B M | T-value | H-B M | 4 | 3 | 2 | L-B M |
|---------------------|-----------|-----------|-----------|-----------|-----------|--------------|--------|--------|--------|--------|--------|
| Small | -0.845*** | -0.516*** | -0.477*** | -0.435*** | -0.579*** | Small | -7.634 | -5.190 | -5.194 | -3.824 | -5.578 |
| 2 | -0.202* | -0.256** | -0.436*** | -0.264** | -0.452*** | 2 | -1.915 | -2.542 | -4.286 | -2.254 | -4.060 |
| 3 | 0.064 | 0.110 | -0.024 | -0.182 | 0.177* | 3 | 0.559 | 0.942 | -0.226 | -1.624 | 1.659 |
| 4 | 0.662*** | 0.359*** | 0.528*** | 0.327*** | 0.391*** | 4 | 5.461 | 3.138 | 4.388 | 2.656 | 3.299 |
| Big | 0.523*** | 0.740*** | 0.373*** | 1.017*** | 0.620*** | Big | 5.051 | 6.587 | 3.310 | 7.966 | 5.857 |
| HML | H-B M | 4 | 3 | 2 | L-B M | T-value | H-B M | 4 | 3 | 2 | L-B M |
| Small | 0.626*** | 0.546*** | 0.243 | -0.361* | -0.208 | Small | 2.978 | 2.892 | 1.395 | -1.668 | -1.056 |
| 2 | 0.481** | 0.587*** | -0.068 | -0.579*** | -0.397* | 2 | 2.398 | 3.065 | -0.354 | -2.606 | -1.878 |
| 3 | 0.500** | 0.321 | -0.010 | -0.238 | -0.321 | 3 | 2.305 | 1.449 | -0.051 | -1.122 | -1.587 |
| 4 | 0.920*** | 0.394* | 0.268 | -0.401* | -0.457** | 4 | 3.996 | 1.816 | 1.169 | -1.716 | -2.026 |
| Big | 0.348* | 1.027*** | 0.033 | -0.593** | -0.662*** | Big | 1.769 | 4.809 | 0.153 | -2.445 | -3.292 |
| RMW | H-B M | 4 | 3 | 2 | L-B M | T-value | H-B M | 4 | 3 | 2 | L-B M |
| Small | -0.598*** | -0.510** | -0.381** | 0.127 | -0.484** | Small | -2.621 | -2.490 | -2.011 | 0.541 | -2.261 |
| 2 | -0.312 | -0.615*** | -0.367* | -0.470* | -0.749*** | 2 | -1.431 | -2.957 | -1.752 | -1.947 | -3.265 |
| 3 | -0.643*** | -0.364 | -0.427** | 0.065 | -0.282 | 3 | -2.732 | -1.510 | -1.972 | 0.283 | -1.285 |
| 4 | -0.657*** | 0.006 | 0.249 | -0.584** | -0.274 | 4 | -2.626 | 0.026 | 1.001 | -2.302 | -1.118 |
| Big | 0.072 | -0.736*** | -0.771*** | -0.562** | -1.096*** | Big | 0.337 | -3.176 | -3.319 | -2.133 | -5.017 |
| CMA | H-B M | 4 | 3 | 2 | L-B M | T-value | H-B M | 4 | 3 | 2 | L-B M |
| Small | 0.133 | 0.198 | 0.220 | 0.443* | 0.169 | Small | 0.518 | 0.860 | 1.037 | 1.684 | 0.704 |
| 2 | 0.235 | 0.424* | 0.406* | 0.444 | 0.397 | 2 | 0.960 | 1.817 | 1.725 | 1.642 | 1.545 |
| 3 | 0.579** | 0.516* | 0.596** | 0.316 | -0.046 | 3 | 2.195 | 1.910 | 2.454 | 1.223 | -0.188 |
| 4 | 0.120 | 0.375 | -0.150 | 0.332 | 0.091 | 4 | 0.429 | 1.417 | -0.540 | 1.169 | 0.331 |
| Big | 0.600** | 0.380 | -0.051 | 0.881*** | 0.534** | Big | 2.506 | 1.460 | -0.197 | 2.984 | 2.182 |
| WML | H-B M | 4 | 3 | 2 | L-B M | T-value | H-B M | 4 | 3 | 2 | L-B M |
| Small | -0.621*** | -0.441*** | -0.430*** | -0.269* | -0.167 | Small | -4.437 | -3.514 | -3.700 | -1.868 | -1.273 |
| 2 | -0.332** | -0.176 | -0.095 | -0.014 | -0.059 | 2 | -2.483 | -1.385 | -0.741 | -0.094 | -0.420 |
| 3 | -0.574*** | 0.016 | 0.111 | -0.033 | -0.272** | 3 | -3.979 | 0.106 | 0.832 | -0.234 | -2.023 |
| 4 | -0.101 | -0.040 | 0.018 | -0.273* | -0.655*** | 4 | -0.661 | -0.277 | 0.116 | -1.755 | -4.366 |
| Big | -0.418*** | -0.259* | 0.294** | -0.675*** | -0.541*** | Big | -3.194 | -1.823 | 2.066 | -4.179 | -4.043 |
| Adj. R ² | H-B M | 4 | 3 | 2 | L-B M | F-value | H-B M | 4 | 3 | 2 | L-B M |
| Small | 0.324 | 0.234 | 0.224 | 0.114 | 0.163 | Small | 25.40 | 16.14 | 15.26 | 6.76 | 10.25 |
| 2 | 0.129 | 0.158 | 0.105 | 0.069 | 0.104 | 2 | 7.80 | 9.90 | 6.19 | 3.92 | 6.15 |
| 3 | 0.158 | 0.074 | 0.072 | 0.050 | 0.059 | 3 | 9.92 | 4.25 | 4.08 | 2.79 | 3.33 |
| 4 | 0.200 | 0.100 | 0.073 | 0.064 | 0.114 | 4 | 13.18 | 5.86 | 4.16 | 3.61 | 6.81 |
| Big | 0.170 | 0.278 | 0.112 | 0.215 | 0.195 | Big | 10.80 | 20.32 | 6.66 | 14.50 | 12.80 |

Notes: Table 11 depicts the findings of momentum adjusted FF5FM using inflation adjusted value-weighted portfolios including intercept, coefficient and adjusted R-square results. The right side shows T-values for individual factor and F-value for 25 portfolios.

Table 12 demonstrate the conclusive results for factor statistically significance in the model. It summarises that out of total 25 portfolios, the number of factors show significant findings using standard APMs. The main market factor (RmInf) shows highly statistically significant results with p-value < 0.05 in all the models. Similarly, size pattern (SMB) shows 21 out of 25 portfolios statistically significant results like (Azam & Naveed, 2021). Moreover, the value pattern (HML) which is assumed redundant in FF5FM internationally also shows stable results and interestingly better performance by augmenting momentum with FF5FM, such as 16 portfolios show significant results while FF5FM shows 13 significant portfolio results. The momentum pattern also shows significant determinant in both models such as C4FM and M-FF5FM, which progressively increase the investment pattern (CMA) significance in the M-FF5FM but inversely influence the profitability pattern (RMW). In conclusion, this study presents strong evidence based on statistically significance of factors out of 25 portfolios in the model which can be concluded as follows:

Table 12: Factor wise Conclusive Significance Results

| FACTOR | CAPM | FF3FM | C4FM | FF5FM | M-FF5FM |
|---------------|------|-------|------|-------|---------|
| RmInf. | 24 | 25 | 25 | 25 | 25 |
| SMB | NA | 21 | 22 | 21 | 21 |
| HML | NA | 12 | 13 | 13 | 16 |
| WML | NA | NA | 12 | NA | 16 |
| RMW | NA | NA | NA | 16 | 9 |
| CMA | NA | NA | NA | 3 | 14 |

Notes: Table 12 summarises the factors performance in the models used, out of overall 25 portfolios such as 24 out of 25 portfolios show significant results for inflation-adjusted market-pattern using CAPM. NA stands for not applied.

GRS Test Results

For further robustness of APMs, this study employed GRS test to examine and discover which APM explains better the average portfolio stocks returns based on inflation-adjusted specification in PSX. GRS detects the appropriate model based on 'mean-variance efficiency of portfolio returns (Brownet al., 2021).

Table 13: Estimates of GRS test

| Model | Mean Alpha | Mean SE | Mean R2 | Mean adj. R2 | GRS F-test | GRS p-value |
|---------|------------|----------|----------|--------------|------------|-------------|
| CAPM | -0.016321 | 0.003752 | 0.032019 | 0.029013 | 2.698162 | 0.000039 |
| FF3FM | -0.015826 | 0.003631 | 0.109683 | 0.101336 | 2.497770 | 0.000152 |
| C4FM | -0.013792 | 0.003883 | 0.123946 | 0.112961 | 1.779121 | 0.013954 |
| FF5FM | -0.015412 | 0.003651 | 0.126527 | 0.112793 | 2.300666 | 0.000565 |
| M-FF5FM | -0.013061 | 0.003873 | 0.142310 | 0.126076 | 1.713144 | 0.020262 |

Notes: Table 13 shows the GRS test results based on asset pricing models which consists of five models. The results show mean alpha, mean standard errors, mean R-square, mean adjusted R-square, GRS F-test and GRS p-value for all models.

Table 13 reports the estimates of (Gibbons et al., 1989) test which hypothesize the sum of intercept should be equal to zero to choose the appropriate model for the market. The findings examine the absolute average alpha (AAA) which indicates how well the combination of factors in the model explains the average portfolio excess returns and the results suggests momentum-augmented FF5FM (AAA = 0.013061, GRS F-test = 1.713144) outperform the remaining baseline models in PSX. This validates that augmenting the number of factors in the CAPM produces appropriate outcomes in the Pakistani equity market. Furthermore, the second suitable model suggested by GRS test is C4FM (AAA = 0.013792, GRS F-test = 1.779121) appropriate in explaining the average portfolio stocks returns in PSX.

Conclusion

The stock market primarily contributes to a country's economic growth by allowing funds to flow smoothly from savers to investors, thereby maintaining the economic cycle and allowing the economy to grow at a steady pace. Asset pricing models have gained considerable popularity, as it is associated with the determination of stock prices volatility in the capital markets. To determine the stock prices, CAPM theory postulates excess returns with respect to surplus over risk-free rates. Based on Fisher (1930) 'theory of interest', this study postulates a novel measurement approach for real excess returns by using inflation adjusted risk-free rates rather than treasury bills rates to calculate both excess portfolios and market returns.

This study presents strong evidence that the momentum augmented (Fama & French, 2015) five-factor model (MFF5FM) equilibrium condition holds in the market. This demonstrates that increasing the number of factors in the APM produces appropriate outcomes in the PSX. Furthermore, the factors such as market, size, and value move largely correlated with excess portfolio stocks prices. Researchers and academicians can use the baseline and momentum augmented FF5FM findings of the study to better comprehend the nexus of risk and returns over a broad stock pattern. Potential investors are given more information about the PSX investment patterns, which is backed up by real-world statistics, before making a decision.

Conclusively, the market portfolios consecutively offer systematic risk premiums in the market. After assuming this study, policy makers can establish investment-friendly policies for domestic as well as foreign investors who are potential investors in future. More importantly, the baseline and momentum augmented FF5FM findings can assist researchers and academicians in understanding the nexus of risk and returns of PSX through a broad stock pattern. Before making a decision, potential investors are provided further information about the PSX investment patterns backed up by real-world data.

The future potential studies in the similar discipline may be feasible if they investigate Covid-19 era as suggested by (Azam & Azeem, 2021); augmenting Human-Capital as additional factor with nested APMs as suggested by (Azam, 2022a); using GDP-Growth as macroeconomic additional mediating variable proposed by (Azam & Naveed, 2021); recently used Tobin-q as additional factor augmented with APMs advocated by (Azam, 2022b); using Leverage as added factor for further robustness of results as recommended by (Azam & Ilyas, 2011) in their studies. Moreover, employing developed and datasets from different nations, it is feasible to use macroeconomic variables and a variety of statistical and econometrical approaches for further robustness.

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Abbreviations list

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| AAA | Absolutely average alpha |
| APMs | Asset pricing models |
| CAPM | Capital Asset Pricing Model |
| C4FM | Carhart (1997) four-factor model |
| FF3FM | Fama & French (1993) three-factor model |
| FF5FM | Fama & French (1993) five-factor model |
| FSD | Financial sector development |
| GRS | Gibbons, Ross & Shanken (1989) test |
| HBMS | High B M ratio stocks portfolios |
| IAREPR | Inflation adjusted real excess portfolio returns |
| LBMS | Low B M ratio stocks portfolios |
| MFF5FM | Momentum augmented FF5FM |
| PSX | Pakistan Stock Exchange |