Journal of Social Sciences and Management Studies
https://doi.org/10.56556/jssms.v1i4.293
ISSN: 2957-8795

RESEARCH ARTICLE

An empirical investigation of Tobin's-Q augmented various Asset Pricing Models: Evidence from Pakistan

Mohammad Azam

Department of Management Sciences, National University of Modern Languages (NUML), Peshawar Campus, KPK, Pakistan

Corresponding Author: Mohammad Azam, azam.ims@hotmail.com; mazam@numl.edu.pk
Received: 17 September, 2022, accepted: 11 October, 2022, Published: 12 October, 2022

Abstract
Despite the strong growing popularity of Asset Pricing Models, it is difficult to estimate which factor contributes significantly in explaining average excess portfolio returns particularly in emerging equity market. Using an extensive sample over Jan-1994-Dec-2020 period, this paper aims to extend the literature by augmenting Tobin-Q adjusted risk premium with various unconditional standard asset pricing models which seeks to postulate the nexus between expected portfolios stock returns and risk-factors using monthly data of 521 enlisted financial and non-financial firms from Pakistan Stock Exchange. The multiple time-series OLS regression analysis models are employed to analyze Tobin-q risk-factor augmented with various factors models. Fama and French (2015) five-factor model excessively explains average equity returns however, our results reveal that size, value, profitability and particularly Tobin-q factor are significant while market and investment factor are redundant in Pakistan Stock Exchange. The momentum factor shows weak results in describing average equity returns in the market. Based on Gibbons, Ross and Shanken (1989) test, our findings support Tobin-Q augmented Fama and French (2015) five-factor model as appropriate for pricing stocks returns in emerging market of Pakistan. The investors, portfolio managers and policy-makers should assume the Tobin-q factor while constructing diversified portfolios for investments in Pakistan Stock Exchange.

Keywords: Tobin-q factor; momentum; profitability; investment; PSX

Introduction and Motivation

In the contemporary era of information, all over the globe, individual investors are eager to invest in stock markets due to sophisticated technology and stringent corporate governance rules concerning transparency of public sector reporting (Lee, Cheng & Chong, 2016). Due to progressive complexity of financial markets, investors must assume numerous factors to increase their earnings (Galankashi, et al., 2020). Consequently, investors are hyperconscious to ascertain and evaluate numerous investments proactively seeking to invest generally into capital markets and particularly into stocks due to easily transferability (liquidity) and excess returns characteristics but conversely high volatility (risks) makes them risk-averse and extremely diligent in stocks and portfolio allocation. Likewise, Škrinjarić (2020) critically outlined the portfolio managers and practitioners' eagerness towards acquiring theoretical knowledge regarding innovative finance and economics theories and sophisticated quantitative techniques in response to investors' quarries in solving their intricate problems to determine the intrinsic value of their investment while investing in stock markets. Although, they assume various theories to determine the stock returns (Ameer & Jamil, 2013) in the quest for secured investment and excess stock returns.
Moreover, as the relative performance in terms of excess average returns, stocks compare to corporate bonds over-perform progressively since one and half century (Madsen, 2003). Therefore, individual investors and financial market practitioners enthusiastically exhibit asymmetric response to determine the intrinsic value of stock (Akhtar, 2021), before constructing diversified portfolios in which the nonsystematic risk could be reduced by accumulating inversely correlated securities. Hence, the portfolio construction and selection of asset is a significant domain in investment management (Wu, et al., 2019; Li et al., 2017). Their eagerness to ascertain the validation of theoretically justified factors which are considered as influential determinants in decision-making process made them realized to empirically examine various asset pricing models (APMs) in real-life decision-making process. Dewandaru et al., (2014); Mengoli (2004) argued that the ultimate determination of investors and portfolio managers

is to outperform than market in a systematic way which might be possible over implementation of empirical research results into the real-life investment strategy.

Penman (1992) argued that accounting fundamental ratios are predominantly used earlier to ascertain the intrinsic value of assets. These sustainability accounting ratios are still using by investors, portfolio managers and particularly policy-makers for decision-making process. Similarly, substantial market ratios such as B|M ratio, price-to-earnings ratio, price-to-dividend ratio were also utilized for evaluation of stocks (Stowe et al., 2007; Bodie et al., 2013) which decreased information asymmetry charges for investors (Alroaia et al., 2012) but subject to economic and environmental differences among multinational firms, accounting information interpretation homogeneously may produce spurious results for investors (Choi et al., 1983). However, investors need to understand the mechanism of multi-markets thoroughly (Akhtar, 2021). However, the investors decisions are also influenced by the characteristics of equity market such as transparency, functionality, efficiency, quick and easy-access to information in the developed equity markets that emerging markets typically lacks (Arikan et al., 2019; Leal et al., 2018).

Besides, investors consider the trade-off between desirable and undesirable thing as endeavor to optimize returns and diminish risk rationally which is possible through portfolio diversification strategy (Markowitz, 1952). As Grobys et al. (2018) comment that inversely correlation between two-factor yields probably higher returns. This doctrine is pioneered by Markowitz as the rule of investor behavior known as portfolio theory which contributed grass-roots to the APMs that triggered unprecedented area of research in financial economics, spurred by a plethora of research studies. However, the notion of investment can be traced back from the contribution of Graham et al. (1934) who postulated the under and over-valuation of assets in the realm of asset returns.

Although, in academic literature of financial economics, the theory of APMs is the most debated prominent discipline, Zada, Rehman, and Khwaja (2018) but still remains inconclusive to determine the standard or empirically appropriate benchmark model among a wide diversity of models. By augmenting various proposed empirically motivated factors, researchers evaluate the pricing abilities of APMs which was independently initiated by the trio (Lintner, 1965); Mossin (1966); (Sharpe, 1964) in the collateral name as Capital Asset Pricing Model (CAPM). It explains the risk and returns trade-off of financial asset which is solely measured by beta (Mukherji et al., 1997) which is assumed as a measure of systematic risk (non-diversifiable risk) in financial economics literature. According to this model, the valuation of asset is possible only through the determinant of market returns and beta is the sole coefficient which measures the risk of an asset, also familiar as single-factor model (SFM) which is widely used for empirically predicting the expected returns and cost of equity, (Situm, 2021; Frank & Shen, 2016; Boyle, 2005).

Furthermore, Sehrawat et al. (2020) comment that risk-diversification is possible by augmenting foreign-securities into portfolios. Feng et al. (2017); Harvey et al. (2016); augmented macro-economic and financial factors to discover the nexus with portfolio returns constructing 25 two-sorted Size-B|M ratio portfolios. Similarly, Rendón (2020) examined foreign exchange risk as additional factor using eight OECD high-income countries daily data. The time span consists of 16 years from Jan-2000 through Feb-2016. Using time-series regression models, the augmented C4FM with regional currency premium (RCP) concluded that global model outperformed than standard C4FM. In the case of Japan and Asian-Pacific markets, the domestic version model demonstrated better results than global version model.

The remainder of the study is organized as section two consists of literature review, section three data and methodology, section four time-series OLS regression, section five results and conclusion of the study.

Literature Review

Over the past five decades, APMs was empirically examined and was used as facilitator for investors, portfolio and fund managers Ali et al. (2018) but later on, a plethora of alternative models were introduced such as intertemporal APMs by Merton (1973), Arbitrage pricing theory (APT) by Ross (1976), liquidity augmented two-factor model by (Liu, 2006). Since 1970s, a plethora of anomalies or factors are empirically investigated by academic researchers and portfolio practitioners that determine the stock returns. The motivation behind the theoretical and empirical examination of various patterns is to improve the explanatory power of the model and focusing on model specification (Davidson et al., 2002). Despite long theoretically and empirically investigation, the preceding literature evidenced dissimilar and unpredictable results regarding the asset pricing models especially in emerging equity markets where investors are hyperconscious to make rational investment decision.

Moreover, various patterns such as earning-to-price ratio (Basu, 1977; Jaffe, Keim & Westerfield, 1989); leverage (Bhandari, 1988); size (Benz, 1981; Basu, 1983); value (Stattman, 1980; Rosenberg et al., 1985) divert the single-factor model into multi-factor model. Among them, the most popular one introduced by (Fama & French, 1992, 1993) who augmented two additional factors named size and value factor that became more popular in the academic literature of finance known as Fama-French three-factor model (henceforth FF3FM). It gained more popularity, assumed as a benchmark in the domains of APMs in finance, and therefore, mostly tested in academic research. They criticized and argued that SFM is not empirically valid in explaining variations in cross-sectional expected returns, therefore, the extended version model with addition to size and value factors appropriately explains the cross-sectional average equity returns.

Moreover, Carhart (1997) hypothesizes the momentum-pattern as additional explanatory variable augmented with FF3FM for enhancing description of average returns which is empirically tested correspondingly and observed improving explanatory power of the model called Carhart (1997) four-factor model (C4FM). Merton (1987); Peng and Xiong (2006) focused on investor behaviour pattern. Subsequently, Fama and French (2015) proposed alternative extended and revised model by further augmenting profitability and investment patterns with the FF3FM which is also empirically examined in a variety of developed and emerging equity markets and supported highly statistically significant results in developed stock markets such as USA, Europe and Asia Pacific (Fama & French, 2015, 2016, 2017). Moreover, Liu et al. (2019) offered alternative value pattern measured by Earnings-Price (E/P) ratio. Huang (2019) using individual stock returns data from 1994 through 2016, argued that these patterns delivered better explanatory power for average stock returns in China. Lin (2017) argues that FF5FM outperform FF3FM consistently in China and particularly value and profitability patters perform better results but investment is redundant in China equity market. Likewise, Nichol and Dowling (2014) argued that profitability pattern exhibited the most potential for UK equity market. Conversely, Kubota and Tekehara (2018) revealed unfavourable results from Japan equity market. Ahlatcıoğlu and Okay (2021) investigated post-earnings announcement drift and comment that FF3FM and FF5FM are unable to explain monthly portfolios returns in Turkey equity market. Similarly, the emerging equity markets also pinpointed poor explanatory findings particularly investment factor such as (Guo et al., 2017) in China equity market and (Zaremba & Czapkiewicz, 2017) in Eastern Europe. Ali et al. (2019) examined four standard asset pricing models like FF3FM, C4FM, FF5FM and momentum augmented FF5FM in PSX.

A general question in asset pricing models area is whether the additional factor improves the explaining ability of the model which can be determined by behaviour of stock returns. With a view to understand the behaviour of stock returns, a plethora of accounting-based and firm-specific measure factors have been proposed and empirically investigated around the globe which exclusively enriched the literature but not yet justified to be conclusive concerning appropriate standard APM which fully capture the stock returns. However, there are various firm-level determinants which explain the variation of portfolio returns which portfolio investors and managers seeking to exploit the forecasting capabilities of APMs by augmenting risk-factors into their opportunistic investments in diversified portfolios (Mosoeu & Kodongo, 2020).

Academically, the APMs are ubiquitously empirical examined research area of finance which is pioneered independently by (Lintner, 1965; Mossin, 1966; Sharpe, 1964) called Single-factor model (SFM). It presumes that beta solely captures the risk of a stock which is thoroughly investigated model shows a broad spectrum of diversified findings and conclusions. It postulates linear nexus between expected returns and risk (beta). This linear relationship is empirically investigated and critically argued by numerous research studies such as Douglas (1969) argued that this nexus is too-flat as a consequence of measurement error in bet, (Miller & Scholes, 1972). To decrease the measurement errors, Black et al. (1972); Fama and MacBeth (1973) interlinked this nexus with portfolio instead of individual firms returns and support the argument that this association is near linear while slope is still under estimated (Fama & French, 1992) also reveal too-flat and statistically insignificant nexus between risk and returns. Moreover, a considerable quantity of research studies has made substantial contributions in the discipline of APM by postulating the determinants of expected stock returns and capturing capability of associated risk premiums. Davidson et al. (2002); Fama and French (1993) argued that firm-specific fundamental variables such as firm size, B|M ratio, dividend yield, momentum capture implicit risk measures which investors demand to be compensated as reward of bearing risks that explain the variation in stock returns, (Pojanavatee, 2020; Shaharuddin et al., 2018; Acheampong & Swanzy, 2016; Lau et al., 2002). Based on beta analysis, there are two versions of CAPM findings such as static OLS where beta coefficient is considered to be constant over time and alternative is time varying beta coefficient. Based on time-varying beta, Kassimatis (2008) reveals that value and momentum patterns react as disappear and conversely, size-factor returns diminishes using Australian equity market data. His findings reveal that momentum pattern is performing poor in the market. Furthermore, the study advocates that as the business cycle (macroeconomic situation) of country to country differs accordingly the return-generating process differs country to country. Therefore, the estimation of beta as constant may cause spurious regression results.

More specifically, there are substantial empirical evidences advocating that FF3FM outperformed single-factor model using developed and emerging equity markets data around the globe. The standard FF3FM has extensively been investigated around the globe using the data of developed stock markets (as Dirkx & Peter, 2020; Kubota & Takehara, 2018; Huynh, 2017) and emerging stock markets (as Mosoeu & Kodongo, 2020; Xie & Qu, 2016). As a matter of fact, empirical results were comparatively improved but FF3FM was generally witnessed incapable in explaining fully cross-sectional stock returns (Novy-Marx, 2013; Mosoeu & Kodongo, 2020). Furthermore, Novy-Marx (2013) and Watanabe et al. (2013) identified an additional pattern to be considered when explaining fluctuations in cross-sectional average stock returns (CSSR) as a profitability factor.

Alternatively, Titman, Wei & Xie (2004) highlighted the significant contribution of investment pattern to explain the variation in CSSR. Furthermore, these two additional patterns were augmented with market, size and value patterns familiar as FF3FM and proposed Fama & French (2015) five-factor model (FF5FM). Similarly, various additional patterns such as liquidity (Racicot & Rentz, 2016, 2017), labour growth rate (Roy & Shijin, 2018), momentum (Dirkx & Peter, 2020; Zheng, Chiang & Nelling, 2020), nominal interest rates (Jareño et al., 2018), growth in macroeconomic (Vassalou, 2003; Li, Vassalou & Xing, 2006), price-to-earnings ratio, dividend yield (Zheng, Chiang & Nelling, 2020) factors were augmented with FF5FM to further improve the explanation of variation in CSSR. These empirical studies still not reached to the conclusion regarding perfectness and the empirical augmentation process is still vigorous in the literature. In the line of zoo-factors, this study seeking to explore the Tobin-q risk premium to price the returns and investigate whether additional factor has explanatory power in emerging equity market of Pakistan. Furthermore, to find out which Tobin-q augmented model do the best job in capturing the average excess portfolio returns. It is pioneered by Kaldor (1966) but later on popularized by (Tobin, 1970). It is used to measure the firm performance, (Wernerfelt & Montgomery, 1988), to operationally evaluate the comparative performance of firms, (Wolfe & Sauaia, 2003) which is used as a proxy for investment opportunities, (Fu, Singhal & Parkash, 2016) to investigate whether stock is overvalued or undervalued. It deals with two variables: the present price of stock measured by statisticians and spot-market price of stock which assumes overvalue outperform undervalue stocks.

The contemporary literature, supported by a diverse range of empirical studies, evidence the robustness of FF5FM estimates explaining expected variation in CSSR (Fama & French, 2015, 2017; Racicot & Rentz, 2016; Zada, et al., 2018; Munir et al., 2020) and capturing risk-premium more vigorously than the preceding APMs in developed and emerging equity markets across the globe. The nexus between multiple factors and portfolio returns has been extensively explored in frontier and emerging equity markets and Pakistan Stock Exchange (PSX) is not exempted [1]. Furthermore, various studies augmented alternative factor to investigate the accelerated performance of the model such as default risk by Khan and Iqbal (2021); liquidity by Racicot, Rentz, and Théoret (2018), momentum by Dirkx and Peter (2020). However, these baseline empirical models evidenced diversified and mix results which cannot be generalized to every equity market simultaneously therefore, this study demonstrates novelty by using Tobin-q augmented CAPM, FF3FM, C4FM and FF5FM for in-depth understanding the new pattern whether TQ is a significant proxy of pricing stocks returns empirically in emerging market of Pakistan.

The progress towards improvement and assessment of appropriate relevant risk-factor or determinant for rationally allocating and empirically testing portfolios made the researchers ultra-cautious which although enhanced the literature of APMs but alternatively severe the problem of specification and identification in the discipline of financial economics. Gospodinov, Kan and Robotti (2019) focus on severe identification problems concerning the nexus between coefficients and risk factors by augmenting irrelevant pattern in the model indicating that spurious results may cause biased conclusion consequently, however, factor allocation must be theoretically adjusted and motivated as well as empirically correlated accordingly. Therefore, in line with APMs, this study postulates a theoretically justified risk-factor known as Tobin-q factor.

Cochrane (2011); Racicot and Rentz (2016) pinpointed that the Q-theory interlinks with stock prices and investment. Furthermore, Hou et al. (2015) stated that by adjusting it can be intertwined the nexus between expected returns and investment. Moreover, this study constructs portfolios and categorizes the firms having higher Tobin-q value outperforms than firms having low Tobin-q value and similarly, we construct factor as firms having higher Tobin-q are overvalued firms and firms having lower Tobin-q are undervalued firms. The Tobin-q is used as measure of firm-value (Zhang et al., 2018). Moreover, based on this mechanism, overvalued portfolio firms outperform undervalued portfolio firms which are symbolized as (OMU) to see the relationship between Tobin-q factor and excess average portfolio returns in PSX.

Nevertheless, practitioners and portfolio managers consistently use Tobin-Q technique to determine the value of an asset while investing in valuable long-terms assets (see Wang & Xiong, 2021; Qin, Luo, & Wang, 2021; He, Shi, Chang, & Wu, 2021; Tsai, Mai & Bui, 2021; Balfoussia & Gibson, 2019). It estimates whether firm is overvalued or undervalued in terms of operational efficiency. This emphasizes the inclination to use Tobin-Q as factor premium where firms having lower Tobin-Q outperform firms having higher Tobin-Q to contribute the "zoo of new factors" argued by (Cochrane, 2011). In this study, we introduced Tobin-Q anomaly as additional factor in association with standard single-factor model and multi-factor standard models like FF3FM, C4FM, FF5FM to

[1] See Khan & Iqbal, 2021; Shah, Shah, Khan, & Ullah, 2021; Munir, Sajjad, Humayon, & Chani, 2020; Chhapra, Rehan, Mirza, & Sohail, 2020; Chhapra, Zehra, Kashif, & Rehan, 2020; Arsalan & Iqbal, 2020; Haqqani, & Rahman, 2020; Zhang, Saqib, Saqib, Mahmood, & Cao, 2019; Haque, & Nasir, 2018; Lohano & Kashif, 2018; Zada et al. (2018); Sadhwani, Bhayo & Bhutto, 2019; Mirza & Reddy, 2017; Hanif, Choudhary & Ismail, 2017; Wu, Imran, Feng, Zhang, & Abbas, 2017; Azam & Ilyas, 2011; Azam (2021); Azam (2022a); Azam (2022b).

keep APMs parsimonious using the financial and non-financial (industrial and non-industrial) sectors firms enlisted on PSX by following (Mirza & Shahid, 2008).

Racicot & Rentz (2016; 2017) examined Paster & Stambaugh (2003) (henceforth PS-2003) liquidity adjusted FF-2015 using improved GMM–based robust instrumental variables technique proposed by Hansen (1982). They used PS-2003 liquidity factor as additional factor using 564 months' time span data from January 1968 to December 2014. They analyzed these APMs using OLS and GMM techniques to investigate the capturing ability of the models. Their findings in both studies reveal that including liquidity factor, all (FF-1993; 2015) factors are unable to explain expected returns except market factor for 12 sectors using sophisticated GMM approach. Conversely, the study also revealed effective results using OLS procedure.

Ali (2022) investigated Hirshleifer and Jiang (2010) zero-investment portfolio as undervalue minus overvalued (UMO) factor as augmented anomaly with base-line models using PSX data from 2013 to 2018. Furthermore, for testing models, maximum squared Sharpe ratio test and GRS are utilized to examine the performance of the augmented base-line models in PSX. The results revealed that UMO performed better and statistically significant in the market. In combination of market, UMO, SMB and RMW patterns, the study observed better models performance as compare to base-line models in the market.

Azimli, (2020) compared various APMs in emerging Borsa Istanbul equity market using monthly stock returns from Jul-2006 through Dec-2015 and Govt. Treasury debt rates are risk-free rate to calculate excess market returns. The annually rebalanced value-weighted six excess portfolios as LHS dependent variables were constructed and regressed against CAPM, FF3FM, Q-factor and FF5FM in this study for investigation. To test the robustness of the model based on joint hypothesis for alpha, GRS and GMM techniques are used.

Khan and Iqbal (2021) investigated FF5FM using non-financial firms' data by employing hybrid version of dynamic panel probit and artificial neural network (ANN) and Fama and MacBeth (1973) two-steps approach to determine the default risk factor in PSX. Using the time span over 2006-2015, they investigated default risk adjusted FF5FM in PSX. Interestingly, the findings reveal that mean stock momentum is highly absorbed by investment factor.

Jan et al. (2021) comparatively examined single and multiple-factor models using 50 firms' data enlisted on PSX for the time-span over Jan-2014-Dec-2018. Surprisingly, their findings reveal contradictory results with theory as big market-cap firms outperform small market-cap. Similarly, low B|M ratio firms outperform high B|M ratio firms. Conclusively, FF3FM performed better results in explaining stock returns than CAPM-single-factor model in PSX.

Davidson, Leledakis and Okunev (2002) examined the impact of Tobin-q ratio on cross-sectional momentum on mean equity returns using individual stock returns data from UK stock market. They used TQ, beta, market value equity (MVE) and B|M equity as (RHS) independent variables. Their findings revealed that MVE and TQ performed highly statistically significant results. Furthermore, firms having smaller TQ outperformed firms having higher TQ. However, Pietrovito (2016) investigated the comparative study of price-to-earnings (P|E) ratios and Tobin's-Q (1969) in investment decision-making process using German equity market firm-level data.

Zada et al. (2019) examined FF5FM in PSX using 16 dynamic portfolios based on size, value, profitability and investment from 120 firms sample monthly data from Jun-2000 through Jun-2014. They observed statistically significant results for five-factors using time-series regression technique except value-factor which show weak significant results. In line with theory, such as small firms outperform big firms, their study revealed all similar as small, value, robust and conservative stocks outperformed big, growth, weak and aggressive stocks respectively. Their findings reveal portfolio constructed on the basis of Big-Low-Robust and Aggressive (BLRA) firms produced efficient results for CAPM, FF3FM and FF5FM with the highest adjusted R-square.

Chhapra et al. (2020) examined the nexus between default risk and CSSR using all enlisted and delisted Pakistani firms' monthly data from 2001-2016. To measure default risk, Ohlson's 0-score is used. For further investigation, CAPM, FF3FM and FF5FM are regressed with equal and value-weighted decile portfolios. Moreover, using GMM for estimation, the findings revealed non-existence of distress anomaly in PSX. Conclusively, the results of B|M ratio revealed statistically insignificant explaining returns of distressed firms.

Shoaib et al. (2020) investigated FF3FM in South Asian three-countries such as China, India and Pakistan using 1,148 firms' panel data for the time-span 2001-2017. They investigated alternative measures of size and revealed that size-effect measures matter for markets as total-assets (TA) is appropriate for emerging (India-Pak) while market-equity based measure for matured markets (China). Furthermore, TA (market-cap-MC) portfolios postulate significant statistical results using FF3FM in all markets. Conversely, market-equity based portfolios postulate weak findings for size-pattern in emerging markets.

Hou et al. (2019) examined various factors models in their work and investigated q-factor includes FF-5 factors and six-factors using spanning factors regression. They further critically argued that mispricing may be due to construction procedure followed.

Hanauer and Linhart (2015) investigated CAPM, FF3FM and C4FM using monthly data from 21 emerging and 24 developed economies. The sample of the study consists of 63,775 unique stocks from developed and 21,612 unique

stocks from emerging equity markets. The time-span assumed consists of 16 years from Jul-1996 through Jun-2012. The stocks prices are measured with US$. Based on 16 value-weighted portfolios, multiple OLS regression technique is used for analysis. Furthermore, mean adjusted R-square, mean absolute intercept, mean standard error of intercepts and F-value of GRS test with p-value are used for model evaluation. Their findings revealed statistically strongly significant value-patterns while weakly but significant momentum-patterns. In comparison with developed markets, the capturing ability of value patterns is observed more influential in emerging markets while poor size-effect is observed in both.
This study contributes novelty to the growing literature of base-line APMs by shedding scholarly new light on the nexus between Tobin-q factor and equity portfolio returns in emerging stock market of Pakistan using augmenting Tobin-q factor as sixth factor to the FF5FM with highly significant findings.

Model Specification

Tobin-Q augmented CAPM (TQ-CAPM)

$$R_i - R_f = R_f + \beta_m(R_m - R_f) + +\beta_q(OmU) + \varepsilon_i \quad (1)$$

Where,
R_i is expected excess return from portfolio
R_f, is Risk-less rate (T-bills rates as proxy)
R_m is the expected return from market
OmU is Overvalued minus Undervalued firms returns called Tobin-Q factor.
β_m and β_q are the sensitivities of market and Tobin-Q factors respectively

Tobin-Q augmented FF3FM (henceforth TQ-FF3FM)

$$R_i - R_f = R_f + \beta_m(R_m - R_f) + \beta_s(SmB) + \beta_v(HmL) + \beta_q(OmU) + \varepsilon_i \quad (2)$$

Where,
R_i is expected excess return from portfolio
R_f, is Risk-less rate (T-bills rates as proxy)
R_m is the expected return from market
R_m-R_f is the excess market returns called Market factor
SmB is the Small Minus Big firms returns called Size factor
HmL is the High minus Low firms returns called Value factor
OmU is Overvalued minus Undervalued firms returns called Tobin-Q factor.
β_m, β_s, β_v and β_q is the coefficients of market, size, value and Tobin-q factors respectively

Tobin-Q augmented Carhart (1997) four-factor model (henceforth TQ-C4FM)

$$R_i - R_f = R_f + \beta_m(R_m - R_f) + \beta_s(SmB) + \beta_v(HmL) + \beta_p(WmL) + \beta_i(OmU) + \varepsilon_i \quad (3)$$

Where,
R_i is expected return from portfolio
R_f, is Risk-less rate (T-bills rates as proxy)
R_m is the expected return from market
R_m-R_f is the excess market returns called Market factor
SmB is the Small minus Big firms returns called Size factor
HmL is the High minus Low firms returns called Value factor
WmL is the Winner minus Loser firms returns called Momentum factor
OMU is Overvalued Minus Undervalued firms returns called Tobin-Q factor.
β_m, β_s, β_v, β_p and β_i are the coefficients of market, size, value, profitability and investment factors respectively

Tobin-Q augmented Fama & French (2015) five-factor model (henceforth TQ-FF5FM)

$$R_i - R_f = R_f + \beta_m(R_m - R_f) + \beta_s(SmB) + \beta_v(HmL) + \beta_p(RmW) + \beta_i(CmA) + \beta_q(OmU) + \varepsilon_i \quad (4)$$

Where,
R_i is expected return from portfolio
R_f, is Risk-less rate (T-bills rates as proxy)
R_m is the expected return from market
R_m-R_f is the excess market returns called Market factor
SMB is the Small Minus Big firms returns called Size factor
HML is the High Minus Low firms returns called Value factor
RMW is the Robust Minus Weak firms returns called Profitability factor
CMA is the Conservative Minus Aggressive firms returns called Investment factor
OMU is Overvalued Minus Undervalued firms returns called Tobin-Q factor.
β_m, β_s, β_v, β_p and β_i are the coefficients of market, size, value, profitability, investment and Tobin-q factors respectively

Data and Methodology

Data and Descriptive Statistics

With a view to conduct empirical analysis, this study uses an extensive historical sample of secondary data, consequently follows the area of quantitative approach, which were extracted from numerous sources of official websites. The data such as closing prices information were collected from Thomson Router DataStream for the time span of 27 years from January 1994 through December

2020. The 3-months Government Treasury Bills rates were used as risk-free rate and financial statements information such as outstanding shares, book-value of equity of individual firms were extracted from State Bank of Pakistan (SBP) official website. Moreover, the monthly data of PSX-100 index (previously called KSE-100 index) were extracted from Pakistan Stock Exchange (PSX) official website.

The population of the study consists of all financial and non-financial firms registered on PSX from 1994 to 2020 which were documented as 564 firms. By following Ali et al. (2018), this study observed 630 firms and based on data availability for consecutive four years, choose 521 firms as a sample for the study while dropped firms having negative book-to-market ratios. The sample of the study covers 521 enlisted individual financial and non-financial firms from all sectors except closed-ended mutual funds. The simple random sampling technique is used for selecting the sample of the study and the firms' data available for consecutive four years were selected for the study while delisted firms were dropped accordingly.

Table 1: Independent variables (factors) description and sources:

Variable	Description and Sources
Market excess Returns (RmRf)	PSX-100 index Close prices. Market returns (Rm) is measured as Price(today) minus Price(yesterday) divided by Price (yesterday) multiply with 100. Excess market returns measured by deducting risk-free rate.
Risk free rate (Rf)	Government 3 months Treasury Bills rate is measured as Risk-free rate.
Size factor (SMB)	Size factor is measure based on market capitalization (market-cap) which is calculated as (market close price multiply with outstanding shares). Small firms having small market-cap are assumed as small market-cap portfolios while big firms having big market-cap are assumed as big market-cap portfolios. Based on market-cap all firms are classified into 5 equal-quantile portfolios. Size factor is measured as small market-cap portfolios minus big market-cap portfolio by following Fama & French (1993, 2015).
Value factor (HML)	Value factor is measured based on Book-value (BV) of equity and Market Close Price of stock which is assumed as Book-to-Market (B|M) ratio. Sorted all the firms' B|M ratio and classified into 5 equal-quantile portfolios similar as size factor and assumed as high B|M ratio firms (value) portfolios minus low B|M ratio firms (growth) portfolios by following Fama & French (1993, 2015).
Profitability factor (RMW)	RMW is calculated using data of operating profitability (OP). OP is the result of subtracting operating profit with interest expense and divided by total shareholder's equity by following (Fama & French, 2015).
Investment factor (CMA)	CMA is calculated using data of investment (Inv). For the portfolio formed in December period t, Inv measured as growth in total assets with equation; $((TA_n - TA_{n-1}) / TA_{n-1}))$ by following (Fama & French, 2015).
Momentum (WML)	Momentum factor is measured by calculating average returns of sample firms for 6 months and then sorted into 5 equal-quantile portfolios as higher average returns (winners) portfolios and lower average returns (losers) portfolios and consequently constructed winners minus losers factors by following (Azimli, 2020).
Tobin's Q (TQ) ratio	TQ is measured as Market value of Common equity plus Total Assets (TA) minus Book-Value (BV) of common equity divided by TA by following (Pietrovito, 2016).

Source: Author's Compilation

Portfolio Construction

To investigate the impact of various factors such as market, size, value, profitability, investment, momentum, liquidity on portfolio stock returns, a substantial quantity of empirically research work has been conducted constructing value-weighted dual-sorted 25 (5x5 sorted) portfolios based on size and B|M ratio. All the firms were equally divided into 5 quintiles equal portfolios based on market capitalization (size) and classified as big, 4, 3, 2 & small firms. After sorting based on highest to lowest market capitalization, these 5 quintiles were further subdivided into 5 equal portfolios based on book-to-market ratios as high B|M ratio firms to low B|M ratio firms as shown in the table-1. Based on size and B|M ratio, 25 value-weighted mimicking portfolios were formed as suggested by FF (1993, 2015) as mentioned in Table 2.

Empirical Analysis

Our model provides an alternative Tobin-q factor as additional augmented factor in the domain of APMs to analyze the impact on average value-weighted excess portfolio returns in PSX. Furthermore, our study differentiates from other studies conducted on PSX using APMs based on construction of value-weighted 25 portfolios which includes financial as well as non-financial enlisted firms excluding Closed-ended Mutual Funds industry. To demonstrate the influence of market, size, value, profitability, investment and Tobin-q factors on average equity portfolio returns, this study observed the following findings:

Table 2: 25 Portfolio Construction based on Size-Book-to-Market (B|M) Ratio

PF-25	H_B|M	4	3	2	L_B|M	PF-25	H_B|M	4	3	2	L_B|M
Big	BH	B4	B3	B2	BL	**Big**	SBM01	SBM02	SBM03	SBM04	SBM05
4	4H	44	43	42	4L	**4**	SBM06	SBM07	SBM08	SBM09	SBM10
3	3H	34	33	32	3L	**3**	SBM11	SBM12	SBM13	SBM14	SBM15
2	2H	24	23	22	2L	**2**	SBM16	SBM17	SBM18	SBM19	SBM20
Small	SH	S4	S3	S2	SL	**Small**	SBM21	SBM22	SBM23	SBM24	SBM25

Table 2 depicts the VW 25 excess portfolios constructed based on Size-B|M ratio. BH represents portfolio consists of firms having big market-cap and high B|M ratios. Similarly, BL represents portfolio consists of firms having big market-cap and low B|M ratios. In the same way, SH indicates the portfolio of small market-cap and high B|M ratios firms and SL shows small market-cap and low B|M ratios financial and non-financial firms. Alternatively, these portfolios are named as SBM1 (Size-B|M ratio) portfolio represents BH and so on.

Descriptive Statistics & Matrix of Correlations between Independent Variables

Table 3: Descriptive Statistics and Correlation for RHS Factors

Factor	RmRf	SMB	HML	WML	RMW	CMA	OMU
Mean	0.00332	-0.00361	-0.00136	0.010484	0.002186	0.00152	-0.00175
Std. Dev.	0.084543	0.033125	0.019417	0.028903	0.016322	0.015593	0.018954
Min	-0.45966	-0.15361	-0.09442	-0.14042	-0.05821	-0.04426	-0.07736
Max	0.235408	0.129705	0.12753	0.195921	0.05481	0.060834	0.064787
Obs.	324	324	324	324	324	324	324
Factor	**RmRf**	**SMB**	**HML**	**WML**	**RMW**	**CMA**	**OMU**
RmRf	1						
SMB	-0.0777	1					
HML	0.011	0.0536	1				
WML	-0.0818	0.231	-0.2476	1			
RMW	0.0172	-0.0307	-0.2464	0.0908	1		
CMA	-0.0018	0.0792	0.2556	0.2684	0.1168	1	
OMU	0.0185	0.3135	-0.0985	-0.1378	-0.0272	0.0248	1

Notes: the table shows the descriptive statistics and correlation matrix for independent (right-hand side) variables. The data observations consist of 324 months' data for market, size, value, profitability, investment and Tobin-Q risk-premiums. The third-sixth rows show average returns, standard deviation, minimum and maximum values of the observations respectively. Similarly, the correlation matrix shows the correlation between independent variables (RmRf, SMB, HML, WML, RMW, CMA and OMU)

Table 3 presents the preliminary descriptive statistics which provide an insight into the independent variables (factors) including market premium (RmRf), size premium (SMB), value premium (HML), momentum premium (WML), profitability premium (RMW), investment premium (CMA) and Tobin-q premium (OMU). On average, market premium exhibits (0.00332) 0.332% monthly average returns with standard deviation of 0.084543 and ranges from -0.459660 to 0.23541.

The correlation matrix results demonstrate that there is inverse nexus between market and size patterns, market and investment; size and profitability; value and profitability, value and Tobin-q, and between profitability and Tobin-q premiums. The results show similarity with Zada et al. (2018) except between SMB & HML, HML & RMW, HML & CMA, and RMW & CMA. In contrast, the remaining premiums show a positive relationship with one another. To the best of our knowledge, Tobin-q has not been used as additional risk-premium factor with FF5FM in the emerging market of Pakistan. Therefore, it will contribute additional factor to the body of knowledge for further investigation of APMs in future.

Table 4 summarizes the descriptive statistics of dependent variables including average 25 excess portfolios returns extracted based on Size-B|M ratio, standard deviation, minimum and maximum values of each value-weighted portfolio.

Table 4: Descriptive Statistics of Dependent Variables (25 Portfolios)

Mean	H_B|M	4	3	2	L_B|M	Std. Dev.	H_B|M	4	3	2	L_B|M
Big	-0.0138	-0.0123	-0.0095	-0.0082	-0.0082	**Big**	0.0711	0.0559	0.0489	0.0603	0.0537
4	-0.0145	-0.0121	-0.0121	-0.0098	-0.0097	**4**	0.0544	0.0522	0.0521	0.0604	0.0568
3	-0.0154	-0.0097	-0.0099	-0.0119	-0.0091	**3**	0.0617	0.0611	0.0503	0.0549	0.0549
2	-0.0135	-0.0129	-0.0088	-0.0131	-0.0179	**2**	0.0681	0.0569	0.0597	0.0648	0.0628
Small	-0.0165	-0.0150	-0.0143	-0.0150	-0.0198	**Small**	0.0536	0.0672	0.0586	0.0742	0.0595
Min	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**Max**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	-0.3436	-0.2891	-0.3145	-0.3372	-0.1713	**Big**	0.1762	0.1426	0.1365	0.3501	0.1688
4	-0.3011	-0.2703	-0.2019	-0.3183	-0.2460	**4**	0.1957	0.1756	0.1374	0.2476	0.1554
3	-0.2549	-0.2728	-0.2851	-0.2114	-0.1892	**3**	0.2591	0.3606	0.1621	0.2600	0.2557
2	-0.3530	-0.2110	-0.1991	-0.2638	-0.2882	**2**	0.3870	0.2053	0.2199	0.2614	0.2415
Small	-0.1748	-0.3308	-0.2150	-0.2999	-0.2873	**Small**	0.2205	0.2600	0.2985	0.2435	0.3085
PF-25	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**PF-25**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	BH	B4	B3	B2	BL	**Big**	SBM01	SBM02	SBM03	SBM04	SBM05
4	4H	44	43	42	4L	**4**	SBM06	SBM07	SBM08	SBM09	SBM10
3	3H	34	33	32	3L	**3**	SBM11	SBM12	SBM13	SBM14	SBM15
2	2H	24	23	22	2L	**2**	SBM16	SBM17	SBM18	SBM19	SBM20
Small	SH	S4	S3	S2	SL	**Small**	SBM21	SBM22	SBM23	SBM24	SBM25

Note: the table 4 shows the descriptive statistics for dependent (left-hand side) variables. The data observations consist of 324 months excess 25 portfolios constructed based on size (market capitalization) and value (book-to-market ratio) using 521 financial and non-financial firms' monthly data. The first group shows average returns, second shows standard deviation, third and fourth minimum and maximum values of the observations respectively for 25 portfolios which are ranked such as BH, B4, B3, B2, BL, 4H, 44, 43, 42, 4L, 3H, 34, 33, 32, 3L, 2H, 24, 23, 22, 2L, SH, S4, S3, S2 and SL. Here, B represents big-firms and S represents small-firms while on the other hand high-book-to-market ratio and low-book-to-market ratio sequence from high to low respectively

In table 4, the portfolio BH (SBM01) represents excess portfolio consists of firms having big market-cap and high B|M ratios minus risk-free rates. It ranges from -0.343553 to 0.1761707 with average returns of -0.01382 and standard deviation of 0.07105. Similarly, portfolio SL (SBM25) denotes excess portfolio having small market-cap and low B|M ratio. It ranges from -0.2872866 to 0.3084587 with average returns of -0.01982 and standard deviation of 0.0595351. Moreover, there is no monotonically movement in the 25 portfolios excess returns when moving from small firms to big firms.

As size theory suggest, the small market-cap firms have greater returns as compare to big market-cap firms but this study presents diversified results. The average returns of all the 25 portfolios show negative returns but on average, 2 small stock quantiles (SBM16-25) shows -0.01467 higher returns with 0.062538 standard deviation in comparison with 2 big stock quantiles (SBM01-10) shows -0.01101 with standard deviation of 0.056565 which supports the theory that small stock produce higher returns as well as higher standard deviation (risk) shows similarity with (Kassimatis, 2011; Gaunt, 2004; Halliwell, Heaney & Sawicki, 1999). However, on average, 2 high B|M ratio stock quantiles (column High-B|M plus 4) exhibits -0.01356 returns with standard deviation of 0.060211 shows lower returns in comparison with 2 Low-B|M ratio stock quantile -0.01226 with standard deviation of 0.060218 which show almost contradictory results.

Factor Redundancy (Spanning) Tests

The spanning test helps to thoroughly identify which factor is redundant (Azimli, 2020) and not supporting in terms of mean-variance efficiency (MVE) to the model. A factor is not robust if its variation is captured by other affiliated factors (Huberman & Kandel, 1987). Alternatively, it identifies that which factor possesses unique information ratio (Ardila-Alvarez et al., 2021) which determines the statistically significant value of intercept of the regression. Fama and French (2017) documented that in factor spanning test in case the intercept value is closed to zero, it implies that dependent variable is redundant to explain average portfolio stocks returns. Table 3 illustrates the results for factor spanning regression tests of six factors of the study. According to our results, OMU exhibits intercept of 0.000809 (t = 0.779) with adjusted R-square of 0.183 shows that OMU cannot be replicated by other pattern. It demonstrates that it is significant to explain average excess portfolio returns in PSX. Similarly, RmRf, HML, RMW and CMA also show insignificant intercept values (0.00457%, 0.00137%, 0.00171%, -0.000558% with t-value = 0.89, 1.369, 1.811, -0.667) respectively. The intercept in RmRf (market premium) regression is positive insignificant (0.00457% with t-value = 0.89). However, market, value, profitability and Tobin-q factor show positive intercepts while investment shows negative statistically insignificant intercept. The SMB (size) and WML (momentum) factors demonstrate significant alpha (α = -0.00587 with t-value = -3.312 with $p < 0.05$) and (α = 0.00913 with t-value = 6.589 with $p < 0.01$) respectively which shows inconsistent results with Guo et al. (2017) and with Europe and Japan of Fama & French (2017). The result shows similarity with North America and Asia Pacific results of Fama & French (2017).

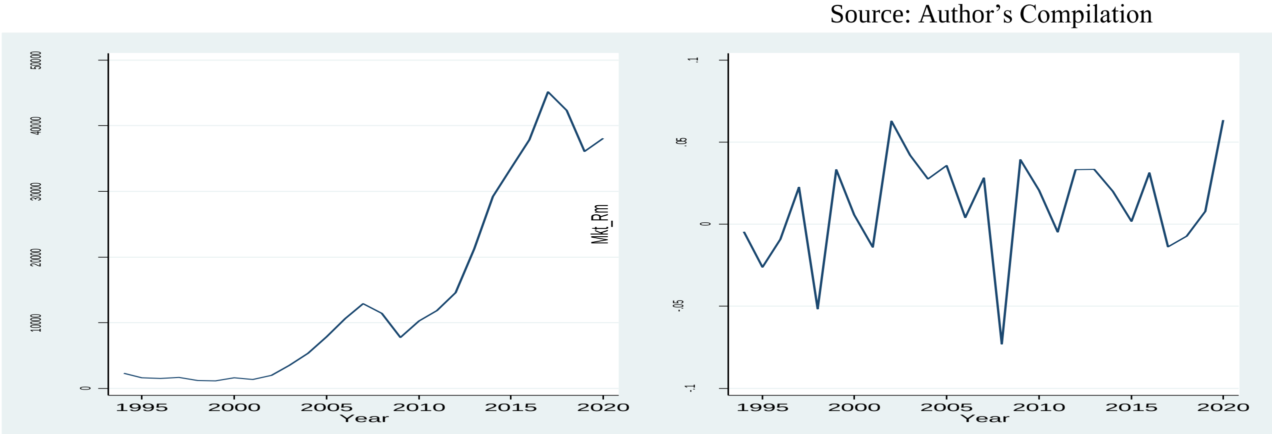
Table 5: Spanning Regression results

Factor	RmRf	SMB	HML	WML	RMW	CMA	OMU
SMB	-0.1876		0.1047***	0.2681***	0.003982	-0.03467	0.2231***
	(-1.173)		(3.422)	(6.162)	(0.135)	(-1.333)	(7.474)
HML	0.009037	0.3402***		-0.5861***	-0.2726***	0.3349***	-0.2499***
	(0.031)	(3.422)		(-7.695)	(-5.354)	(7.778)	(-4.413)
WML	-0.1945	0.3990***	-0.2686***		-0.03299	0.2122***	-0.2127***
	(-0.996)	(6.162)	(-7.695)		(-0.918)	(7.196)	(-5.650)
RMW	0.1047	0.01443	-0.3042***	-0.08031		0.1771***	-0.07819
	(0.343)	(0.135)	(-5.354)	(-0.918)		(3.642)	(-1.272)
CMA	0.09827	-0.1609	0.4785***	0.6617***	0.2268***		0.1876**
	(0.285)	(-1.333)	(7.778)	(7.196)	(3.642)		(2.721)
OMU	0.146	0.6715***	-0.2316***	-0.4301***	-0.06494	0.1217**	
	(0.525)	(7.474)	(-4.413)	(-5.650)	(-1.272)	(2.721)	
RmRf		-0.02303	0.0003416	-0.01604	0.003547	0.0026	0.005955
		(-1.173)	(0.031)	(-0.996)	(0.343)	(0.285)	(0.525)
_cons	0.00457	-0.00587**	0.00137	0.00913***	0.00171	-0.000558	0.000809
	(0.89)	(-3.312)	(1.369)	(6.589)	(1.811)	(-0.667)	(0.779)
N	324	324	324	324	324	324	324
adj. R-sq	-0.007	0.195	0.279	0.29	0.085	0.217	0.183
F	0.6468	14.038	21.799	22.949	6.0045	15.931	13.066

t-stat. are in parenthesis
**** p<0.01, ** p<0.05, * p<0.1*

Using Tobin-q and momentum augmented five-factors to regress the seven factor using data from Jan-1994 to Dec-2020 which consists of 324 months.

Figure 1: Average annual PSX-100 Index Data and Average annual Market returns (1994-2020)
Source: Author's Compilation



As the figure 1 depicts that PSX-index shows steadily inclines in value from 2003-2008 and suddenly declines during financial crises 2008-2009. After 2009, again inclines towards growth while reached to peak of 45,135.94 in 2017 but again slight decline is shown due to political uncertainity in the country. It shows the overall journey from 1994-2020 at a glance to understand the market situation of the country. However, annual market-returns show variation and particularly in 2008-09 era, a big volatility is reflected but at the end 2019-2020 positive growth is shown in the graph.

Figure 1 illustrates the graphical presentation of the PSX-100 index from 1994 through 2020. It shows slowly and consistently growth till 2008 but due to global economic crisis of 2008-2009 influence adversely while after that again a growing increase till Aug-2017 which showed recorded decline abruptly due to bulk-selling of blue-chips stocks by financial institutions and foreign investors and particularly the verdict on 'Avenfield' case of Mr. Nawaz Sharif (X-Prime Minister of Pakistan). Hence, government need to perform their role in the economy as it is a significant agent in the economy (Vasilev, 2021). Moreover, due to current government bold decisions, the investors' confidence level in investing stocks market again increased which ultimately reflect in the shape of bullish trend in the market in 2019 onwards. The similar portray is presented by average returns graph which reflect the same mechanism in the Pakistan equity market.

OLS Regression Results

The additional information content of Tobin-Q as factor, alternative estimator is used to augment the base-line asset pricing models, results demonstrate as follows:

Table 6: Tobin Q augmented CAPM: (Based on Size and B|M Ratio-Double sorted)

Alpha	H_B|M	4	3	2	L_B|M	P-value	H_B|M	4	3	2	L_B|M
Big	-0.0131***	-0.0118***	-0.0094***	-0.0068*	-0.0085**	**Big**	-0.001	0.000	-0.001	-0.036	-0.005
4	-0.0137***	-0.0112***	-0.0116***	-0.0094**	-0.0090**	**4**	0.000	0.000	0.000	-0.006	-0.005
3	-0.0135***	-0.0086*	-0.0099***	-0.0116***	-0.0083**	**3**	0.000	-0.010	-0.001	0.000	-0.006
2	-0.0117**	-0.0127***	-0.0075*	-0.0114**	-0.0160***	**2**	-0.002	0.000	-0.022	-0.001	0.000
Small	-0.0158***	-0.0135***	-0.0125***	-0.0118**	-0.0178***	**Small**	0.000	0.000	0.000	-0.001	0.000
RmRf	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**P-value**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	0.0818	0.0402	0.0612	0.053	0.0493	**Big**	-0.077	-0.272	-0.056	-0.166	-0.164
4	0.0433	0.029	0.027	0.018	-0.0068	**4**	-0.219	-0.391	-0.431	-0.651	-0.854
3	0.0151	0.0425	0.014	0.0199	0.0359	**3**	-0.693	-0.280	-0.675	-0.582	-0.314
2	-0.0162	0.0309	-0.0764*	0.0182	0.0419	**2**	-0.707	-0.409	-0.048	-0.656	-0.283
Small	0.0331	0.0031	-0.0037	0.0261	0.016	**Small**	-0.344	-0.943	-0.920	-0.541	-0.659
OMU	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**P-value**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	0.5396**	0.3294*	0.1877	0.8661***	-0.1203	**Big**	-0.009	-0.044	-0.189	0.000	-0.446
4	0.5487***	0.5684***	0.2941	0.2734	0.3408*	**4**	-0.001	0.000	-0.055	-0.124	-0.041
3	1.1395***	0.6891***	0.005	0.1983	0.5120**	**3**	0.000	0.000	-0.973	-0.220	-0.001
2	1.0343***	0.1851	0.5775***	1.0306***	1.1150***	**2**	0.000	-0.269	-0.001	0.000	0.000
Small	0.4520**	0.8189***	1.0155***	1.9082***	1.1988***	**Small**	-0.004	0.000	0.000	0.000	0.000
Adj. R^2	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**F-value**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	0.025	0.01	0.011	0.075	0.002	**Big**	5.09	2.68	2.75	14.04	1.24
4	0.036	0.039	0.007	0.002	0.007	**4**	6.96	7.58	2.20	1.31	2.11
3	0.118	0.044	-0.006	0	0.029	**3**	22.52	8.39	0.09	0.92	5.77
2	0.077	0	0.039	0.086	0.112	**2**	14.55	0.97	7.50	16.22	21.28
Small	0.023	0.047	0.102	0.234	0.141	**Small**	4.72	9.05	19.37	50.46	27.55

Note: Table 6 reports the time-series OLS regression results for Tobin-Q augmented CAPM. The LHS variables are excess 25 value-weighted portfolios constructed based on size and B|M ratio. The RHS variables are market excess returns and Tobin-Q factor premium. The results reports coefficients with probability values and F-values for all 25 regression results.

First, this study conducts OLS time-series regression using TQ-CAPM Equation-1. According to the methodology elaborated above, Table 6 shows Tobin-Q (TQ) factor augmented CAPM (henceforth TQ-CAPM) time-series OLS regression results, the market premium shows statistically positive but insignificant nexus with portfolio (PF) excess returns except one portfolio (23, β = -0.0764 with p-value = 0.0481), show similarity with (Shah et al., 2021; Rizwan et al., 2013; Hanif & Bhatti, 2010; Javid, & Ahmad, 2008) while dissimilar with Azam and Ilyas (2011) who observed significant and positive nexus.

Furthermore, results reveal positive relationship between market premium and excess portfolio returns except portfolios (4L, 2H, 23 and S3) with β = -0.0068, -0.0162, -0.0764 and -0.0037 respectively). Conclusively, the market-risk premium shows economically insignificant nexus with portfolio stocks returns in PSX using Tobin-Q adjusted CAPM. Conversely, the Tobin-Q pattern demonstrates almost statistically significant nexus with average excess portfolio stock returns except seven portfolios (B3, BL, 43, 42, 33, 32 and 24 with p-value > 0.05). All the portfolios show positive nexus except portfolio BL (β = -0.1203). However, considering TQ as additional independent RHS factor, the results change drastically.

Monotonically, the small market-cap firms' portfolios demonstrate statistically strong significant nexus with portfolio returns except portfolio 24, (β = 0.185 with p-

value = 0.2685) which confirms the theory that small firms outperform big firms based on Tobin-Q anomaly. Similarly, portfolio having H_B|M ratio show statistically significant relationship for all portfolios except PF-24 which also supports the theory that value stocks portfolios outperform growth stocks portfolios.
The TQ-CAPM all alpha coefficients demonstrate statistical highly significant results which designate over-valued portfolios which also determine the non-validity of TQ-CAPM. Interestingly, the F-value of 9 portfolios shows statistically insignificant results while 16/25 portfolios show significant overall models results. The adjusted R-square (ARS) ranges from -0.00 to 23.4%. The average ARS of all 25 portfolios is 5.04% based on measuring goodness of fit.

Table 7: OMU (Tobin-Q) adjusted FF3FM (TQ-FF3FM)

Alpha	H_B|M	4	3	2	L_B|M	P-value	H_B|M	4	3	2	L_B|M
Big	-0.0146***	-0.0125***	-0.0104***	-0.0089**	-0.0104***	Big	0.000	0.000	0.000	-0.003	0.000
4	-0.0137***	-0.0110***	-0.0130***	-0.0110***	-0.0108***	4	0.000	0.000	0.000	-0.001	0.000
3	-0.0127***	-0.0080*	-0.0098***	-0.0127***	-0.0086**	3	0.000	-0.017	-0.001	0.000	-0.005
2	-0.0088**	-0.0113***	-0.0063	-0.0113**	-0.0161***	2	-0.009	0.000	-0.051	-0.001	0.000
Small	-0.0140***	-0.0101**	-0.0118***	-0.0102**	-0.0171***	Small	0.000	-0.002	0.000	-0.004	0.000
RmRf	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**P-value**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	0.0366	0.0104	0.0352	0.0276	0.0256	Big	-0.313	-0.736	-0.204	-0.428	-0.427
4	0.0247	0.0092	0.0059	0.006	-0.0276	4	-0.449	-0.760	-0.852	-0.878	-0.431
3	-0.0003	0.0384	0.0117	0.0102	0.0351	3	-0.994	-0.325	-0.725	-0.776	-0.328
2	-0.0063	0.0387	-0.0646	0.0193	0.0416	2	-0.873	-0.292	-0.089	-0.638	-0.290
Small	0.0419	0.0157	0.0026	0.0436	0.0251	Small	-0.211	-0.677	-0.944	-0.294	-0.487
SMB	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**P-value**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	-1.2805***	-0.8271***	-0.7444***	-0.7830***	-0.7244***	Big	0.000	0.000	0.000	0.000	0.000
4	-0.4954***	-0.5179***	-0.6287***	-0.4011***	-0.6437***	4	0.000	0.000	0.000	0.000	0.000
3	-0.3706***	-0.077	-0.0566	-0.3134**	-0.0385	3	0.000	-0.464	-0.531	-0.001	-0.690
2	0.4096***	0.2799**	0.3783***	0.0324	-0.0129	2	0.000	-0.005	0.000	-0.770	-0.903
Small	0.3252***	0.5129***	0.2021*	0.5437***	0.2751**	Small	0.000	0.000	-0.041	0.000	-0.005
HML	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**P-value**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	1.1308***	0.9218***	0.5690***	-0.085	-0.015	Big	0.000	0.000	0.000	-0.576	-0.915
4	0.7856***	0.9367***	0.1613	-0.3742*	-0.1003	4	0.000	0.000	-0.248	-0.028	-0.513
3	1.0975***	0.5454**	0.1239	-0.173	-0.1485	3	0.000	-0.002	-0.398	-0.269	-0.344
2	1.1857***	0.4569**	0.1829	-0.0326	-0.0398	2	0.000	-0.005	-0.271	-0.856	-0.817
Small	0.6166***	1.3929***	0.1332	0.1077	-0.0406	Small	0.000	0.000	-0.405	-0.553	-0.798
OMU	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**P-value**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	1.3590***	0.8780***	0.6551***	1.2886***	0.277	Big	0.000	0.000	0.000	0.000	-0.069
4	0.9009***	0.9483***	0.6566***	0.4564*	0.6851***	4	0.000	0.000	0.000	-0.013	0.000
3	1.4545***	0.7866***	0.0486	0.3533*	0.5182**	3	0.000	0.000	-0.758	-0.037	-0.002
2	0.9287***	0.0772	0.3877*	1.0095***	1.1180***	2	0.000	-0.656	-0.031	0.000	0.000
Small	0.3353*	0.6773***	0.9177***	1.6198***	1.0432***	Small	-0.034	0.000	0.000	0.000	0.000
Adj. R^2	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**F-value**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	0.405	0.298	0.267	0.24	0.177	Big	56.02	35.22	30.37	26.43	18.31
4	0.176	0.233	0.145	0.059	0.132	4	18.28	25.48	14.72	6.03	13.25
3	0.255	0.068	-0.009	0.031	0.026	3	28.70	6.92	0.30	3.62	3.16
2	0.234	0.046	0.078	0.081	0.106	2	25.65	4.93	7.86	8.09	10.60
Small	0.111	0.279	0.111	0.285	0.157	Small	11.05	32.30	11.09	33.19	16.02

Note: Table 7 reports the time-series OLS regression results for Tobin-Q augmented FF3FM. The LHS variables are excess 25 value-weighted portfolios constructed based on size and B|M ratio. The RHS variables are market excess returns and Tobin-Q factor premium. The results reports coefficients with probability values and F-values for all 25 regression results.

Table 8: OMU (Tobin Q) adjusted Carhart (1997) four-factor model (TQ-C4FM)

Alpha	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**P-value**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	-0.0109***	-0.0099***	-0.0075**	-0.0097**	-0.0095**	**Big**	-0.001	-0.001	-0.003	-0.002	-0.001
4	-0.0125***	-0.0120***	-0.0143***	-0.0128***	-0.0124***	**4**	0.000	0.000	0.000	0.000	0.000
3	-0.0106**	-0.0110**	-0.0125***	-0.0139***	-0.0068*	**3**	-0.001	-0.002	0.000	0.000	-0.039
2	-0.0098**	-0.0120***	-0.0072*	-0.0111**	-0.0114**	**2**	-0.007	0.000	-0.038	-0.003	-0.001
Small	-0.0116***	-0.0094**	-0.0168***	-0.0079*	-0.0144***	**Small**	0.000	-0.007	0.000	-0.036	0.000
RmRf	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**P-value**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	0.0305	0.0062	0.0306	0.0288	0.0242	**Big**	-0.394	-0.839	-0.263	-0.408	-0.454
4	0.0227	0.0107	0.008	0.0089	-0.0251	**4**	-0.487	-0.724	-0.803	-0.818	-0.474
3	-0.0037	0.0432	0.0161	0.0122	0.0321	**3**	-0.915	-0.266	-0.628	-0.734	-0.371
2	-0.0046	0.0399	-0.0631	0.019	0.034	**2**	-0.906	-0.279	-0.097	-0.645	-0.378
Small	0.038	0.0147	0.0106	0.0399	0.0207	**Small**	-0.254	-0.697	-0.767	-0.336	-0.565
SMB	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**P-value**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	-1.1756***	-0.7548***	-0.6649***	-0.8046***	-0.7003***	**Big**	0.000	0.000	0.000	0.000	0.000
4	-0.4608***	-0.5431***	-0.6641***	-0.4518***	-0.6877***	**4**	0.000	0.000	0.000	0.000	0.000
3	-0.3105**	-0.1602	-0.1316	-0.3478***	0.0137	**3**	-0.002	-0.147	-0.165	-0.001	-0.893
2	0.3807***	0.2605*	0.3530**	0.038	0.1174	**2**	-0.001	-0.013	-0.001	-0.746	-0.286
Small	0.3929***	0.5306***	0.0634	0.6070***	0.3524***	**Small**	0.000	0.000	-0.533	0.000	-0.001
HML	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**P-value**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	0.9714***	0.8119***	0.4484***	-0.0522	-0.0516	**Big**	0.000	0.000	0.000	-0.744	-0.728
4	0.7331***	0.9749***	0.215	-0.2973	-0.0334	**4**	0.000	0.000	-0.142	-0.095	-0.835
3	1.0063***	0.6717***	0.2378	-0.1207	-0.2278	**3**	0.000	0.000	-0.119	-0.463	-0.166
2	1.2295***	0.4864**	0.2213	-0.041	-0.2376	**2**	0.000	-0.004	-0.204	-0.828	-0.180
Small	0.5137***	1.3660***	0.3438*	0.0115	-0.1579	**Small**	-0.001	0.000	-0.036	-0.952	-0.339
WML	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**P-value**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	-0.3681**	-0.2538*	-0.2786**	0.0757	-0.0845	**Big**	-0.002	-0.011	-0.002	-0.501	-0.418
4	-0.1213	0.0882	0.124	0.1776	0.1545	**4**	-0.251	-0.368	-0.229	-0.156	-0.172
3	-0.2105	0.2916*	0.2631*	0.1208	-0.1833	**3**	-0.064	-0.020	-0.015	-0.296	-0.113
2	0.1012	0.0682	0.0888	-0.0196	-0.4569***	**2**	-0.428	-0.567	-0.469	-0.883	0.000
Small	-0.2375*	-0.062	0.4862***	-0.2223	-0.2709*	**Small**	-0.028	-0.612	0.000	-0.097	-0.020
OMU	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**P-value**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	1.2085***	0.7743***	0.5412***	1.3195***	0.2424	**Big**	0.000	0.000	0.000	0.000	-0.126
4	0.8514***	0.9844***	0.7073***	0.5290**	0.7482***	**4**	0.000	0.000	0.000	-0.006	0.000
3	1.3684***	0.9058***	0.1561	0.4027*	0.4433*	**3**	0.000	0.000	-0.337	-0.022	-0.012
2	0.9701***	0.105	0.4240*	1.0014***	0.9313***	**2**	0.000	-0.560	-0.023	0.000	0.000
Small	0.2383	0.6520***	1.1164***	1.5290***	0.9325***	**Small**	-0.144	-0.001	0.000	0.000	0.000
Adj. R^2	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**F-value**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	0.422	0.31	0.287	0.238	0.176	**Big**	48.14	29.98	26.98	21.20	14.76
4	0.177	0.232	0.146	0.062	0.134	**4**	14.90	20.54	12.08	5.25	11.00
3	0.261	0.081	0.007	0.032	0.031	**3**	23.82	6.70	1.45	3.11	3.05
2	0.233	0.044	0.077	0.078	0.14	**2**	20.62	4.00	6.39	6.46	11.50
Small	0.121	0.278	0.156	0.289	0.168	**Small**	9.93	25.83	12.91	27.25	14.09

Note: Table 8 reports the time-series OLS regression results for Tobin-Q augmented C4FM. The LHS variables are excess 25 value-weighted portfolios constructed based on size and B|M ratio. The RHS variables are market excess returns and Tobin-Q factor premium. The results reports coefficients with probability values and F-values for all 25 regression results.

Second, this study conducts OLS time-series regression using TQ-FF3FM Equation-2. Table 7 summarizes the results of Tobin-Q adjusted FF3FM (TQ-FF3FM). The results demonstrate that market premium depicts statistically insignificant for all 25 portfolios consistent with TQ-CAPM. This relationship shows positive nexus except portfolios (4L, 3H, 2H and 23 having β = -0.0276, -0.0003, -0.0063 and -0.0646 respectively) which show inverse nexus with portfolio excess returns. The results show the size-pattern statistically insignificant impact on portfolio returns for 5/25 portfolios.

The results demonstrate almost similar findings as TQ-FF3FM show 20/25 portfolios statistically highly significant estimates of slope coefficients concerning size-pattern. Shoaib, Siddiqui & Ayub (2020) argued that positive and significant size infers that small market-cap (MC) firms outperform big MC firms. The findings also establish positive significant for small MC firms except (PF 22 and 2L, β = 0.0324 and -0.0129 respectively). The positive statistically significant value-effect assures the existence of value-effect which infers that value firms outperform growth firms having negative coefficient values except PF-42 (β = -0.3742). Similarly, TQ-pattern shows highly statistically significant nexus with portfolio excess stock returns (PESR) as 22/25 portfolios coefficients exhibit significant coefficients. Comparatively, TQ-FF3FM perform better than TQ-CAPM in the line of TQ-pattern. Conversely, the market risk premium shows similarly statistically insignificant results as TQ-CAPM. The average adjusted R-square (AAR2) shows 0.0504 and 0.15964 for TQ-CAPM and TQ-FF3FM respectively which shows better improvement. The F-state also confirms that 22/25 portfolios exhibit highly significant results with p-value less than threshold value.

Third, this study conducts OLS time-series regression using TQ-C4FM Equation-3. Table 8 summarizes the findings of the Tobin-q augmented Carhart (1997) four-factor model (TQ-C4FM). The market risk premium shows statistically positive insignificant results as TQ-CAPM and TQ-FF3MF. Similarly, the size, value and TQ results exhibit statistically significant as 19/25, 12/25 and 21/25 portfolios respectively while momentum-pattern shows weakly significant findings as 9/25 portfolios. The F-value shows significant findings except 3 portfolios, almost similar to TQ-FF3FM which shows trivial improvement in the TQ-C4FM. Conclusively, the AAR2 for TQ-FF3FM and TQ-C4FM show 0.15964 and 0.1672 respectively which also demonstrates trivial improvement. Interestingly, all the intercept values show statistically significant results.

Table 9: OMU (Tobin Q) adjusted Fama & French (2015) five-factor model (TQ-FF5FM)

Alpha	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**P-value**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	-0.0132***	-0.0115***	-0.0097***	-0.0098**	-0.0097***	**Big**	0.000	0.000	0.000	-0.001	-0.001
4	-0.0133***	-0.0106***	-0.0130***	-0.0110***	-0.0102***	**4**	0.000	0.000	0.000	-0.001	-0.001
3	-0.0121***	-0.0084*	-0.0104***	-0.0134***	-0.0078*	**3**	0.000	-0.012	0.000	0.000	-0.012
2	-0.0078*	-0.0120***	-0.0065*	-0.0106**	-0.0151***	**2**	-0.022	0.000	-0.046	-0.003	0.000
Small	-0.0148***	-0.0092**	-0.0107***	-0.0102**	-0.0156***	**Small**	0.000	-0.004	-0.001	-0.004	0.000
RmRf	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**P-value**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	0.0384	0.0121	0.0364	0.0267	0.0274	**Big**	-0.284	-0.693	-0.187	-0.442	-0.392
4	0.0257	0.0115	0.0073	0.0078	-0.0247	**4**	-0.433	-0.701	-0.819	-0.839	-0.471
3	0.0018	0.0397	0.0136	0.0098	0.036	**3**	-0.958	-0.306	-0.678	-0.783	-0.315
2	-0.0039	0.0387	-0.0657	0.0214	0.0421	**2**	-0.920	-0.292	-0.084	-0.601	-0.282
Small	0.0415	0.0185	0.0055	0.0453	0.0292	**Small**	-0.214	-0.619	-0.879	-0.273	-0.401
SMB	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**P-value**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	-1.2719***	-0.8238***	-0.7422***	-0.7886***	-0.7256***	**Big**	0.000	0.000	0.000	0.000	0.000
4	-0.4949***	-0.5247***	-0.6369***	-0.4119***	-0.6520***	**4**	0.000	0.000	0.000	0.000	0.000
3	-0.3741***	-0.0897	-0.0739	-0.3208***	-0.0323	**3**	0.000	-0.392	-0.404	-0.001	-0.738
2	0.4094***	0.2708**	0.3821***	0.0297	-0.003	**2**	0.000	-0.007	0.000	-0.788	-0.977
Small	0.3168***	0.5083***	0.1998*	0.5341***	0.2718**	**Small**	-0.001	0.000	-0.040	0.000	-0.004
HML	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**P-value**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	1.0979***	0.8569***	0.5201***	-0.0789	-0.1218	**Big**	0.000	0.000	0.000	-0.631	-0.421
4	0.7406***	0.7671***	0.029	-0.5527**	-0.3162	**4**	0.000	0.000	-0.847	-0.003	-0.052
3	0.9591***	0.3787*	-0.1044	-0.2108	-0.148	**3**	0.000	-0.039	-0.500	-0.213	-0.382
2	1.0585***	0.3891*	0.2739	-0.1616	0.0046	**2**	0.000	-0.026	-0.127	-0.403	-0.980
Small	0.5762***	1.2112***	-0.0395	-0.057	-0.2814	**Small**	0.000	0.000	-0.816	-0.771	-0.088
RMW	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**P-value**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	-0.4714*	-0.4235*	-0.3084*	0.2349	-0.4495*	**Big**	-0.016	-0.012	-0.040	-0.214	-0.010
4	-0.2277	-0.5369**	-0.3137	-0.4337*	-0.6949***	**4**	-0.200	-0.001	-0.070	-0.039	0.000
3	-0.5109**	-0.3072	-0.4237*	0.0984	-0.229	**3**	-0.008	-0.145	-0.018	-0.612	-0.239
2	-0.5838**	0.0221	0.2833	-0.4976*	-0.1576	**2**	-0.006	-0.912	-0.170	-0.026	-0.459
Small	0.1232	-0.6743***	-0.7194***	-0.4135	-0.9962***	**Small**	-0.497	-0.001	0.000	-0.066	0.000
CMA	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**P-value**	**H_B|M**	**4**	**3**	**2**	**L_B|M**

Big	-0.3253	-0.1207	-0.0799	0.2125	0.0566	**Big**	-0.112	-0.491	-0.611	-0.284	-0.757
4	-0.0156	0.2716	0.3205	0.4218	0.3348	**4**	-0.933	-0.111	-0.077	-0.056	-0.088
3	0.1471	0.4946*	0.6742***	0.285	-0.2383	**3**	-0.461	-0.026	0.000	-0.163	-0.243
2	0.0174	0.3526	-0.1512	0.1155	-0.378	**2**	-0.938	-0.093	-0.485	-0.621	-0.092
Small	0.3234	0.1893	0.0994	0.3755	0.1456	**Small**	-0.090	-0.373	-0.628	-0.112	-0.464
OMU	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**P-value**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	1.3463***	0.8621***	0.6433***	1.2935***	0.2551	**Big**	0.000	0.000	0.000	0.000	-0.093
4	0.8910***	0.9166***	0.6338***	0.4254*	0.6445***	**4**	0.000	0.000	0.000	-0.020	0.000
3	1.4273***	0.7594***	0.0113	0.3501*	0.5142**	**3**	0.000	0.000	-0.942	-0.039	-0.003
2	0.9018***	0.0687	0.4047*	0.9838***	1.1211***	**2**	0.000	-0.692	-0.025	0.000	0.000
Small	0.3322*	0.6417***	0.8824***	1.5909***	0.9941***	**Small**	-0.036	0.000	0.000	0.000	0.000
Adj. R^2	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**F-value**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	0.42	0.31	0.274	0.243	0.189	**Big**	40.04	25.24	21.30	18.26	13.51
4	0.176	0.256	0.154	0.072	0.165	**4**	12.47	19.51	10.82	5.20	11.67
3	0.268	0.08	0.034	0.033	0.03	**3**	20.67	5.69	2.87	2.85	2.69
2	0.248	0.05	0.079	0.089	0.112	**2**	18.71	3.81	5.59	6.28	7.79
Small	0.116	0.299	0.142	0.292	0.22	**Small**	8.07	24.02	9.94	23.17	16.20

Note: Table 9 reports the time-series OLS regression results for Tobin-Q augmented FF5FM. The LHS variables are excess 25 value-weighted portfolios constructed based on size and B|M ratio. The RHS variables are market excess returns and Tobin-Q factor premium. The results reports coefficients with probability values and F-values for all 25 regression results.

Fourth, this study conducts OLS time-series regression using TQ-FF5FM Equation-4. Table 9 summarizes the TQ-FF5FM findings received from time-series regression analysis. The results show similarity as TQ-CAPM, TQ-FF3FM, TQ-C4FM and TQ-FF5FM, as far as the market risk premium is concerned which shows 25/25 portfolios statistically insignificant and mostly positive nexus except three portfolios (4H, 2L & 23, β = -0.0247, -0.0039 & -0.0657 respectively) with excess portfolio stocks returns (EPSR). Conversely, size-pattern presents 20/25 portfolios economically significant nexus with EPSR. The theory suggests small firms PF outperform big firms PF and findings of small firms demonstrate positive statistically significant returns while big firms PF exhibit negative statistically returns which supports the theory and size-pattern existence in the market. Similarly, HML-pattern shows almost similar results as TQ-FF3FM and QT-C4FM and all three models displays 12/25 statistically significant nexus with EPSR. The high book-to-market portfolios (H-BMPF) exhibits economically positive significant nexus while low book-to-market portfolios (L-BMPF) show insignificant which confirms the existence of value-effect. Moreover, similar to Ekaputra, & Sutrisno, 2020, the result of HML (value) pattern is not redundant for PSX. The profitability-pattern displays 14/25 statistically significant impact on EPSR which indicates significant contribution in the emerging market. Conversely, the investment-pattern coefficient values display highly insignificant and mix results as 23/25 show insignificant. Only 2 PF (34 and 33 having β = 0.4946 and 0.6742 respectively) present significant results. TQ-pattern maintains the significance in the model as 22/25 portfolios show statistically significant and positive nexus with EPSR.

Benchmark Factor

Table 10: Significant or Insignificant Factor Results

Model	RmRf	OMU	SMB	HML	WML	RMW	CMA
TQ-CAPM	1/25	18/25					
TQ-FF3FM	0/25	22/25	20/25	12/25			
TQ-C4FM	0/25	21/25	19/25	12/25	9/25		
TQ-FF5FM	0/25	22/25	20/25	12/25		14/25	2/25
Results	**Insig.**	**Sig.**	**Sig.**	**Sig.**	**Insig.**	**Sig.**	**Insig.**

Note: Table 10 reports the time-series OLS regression results for Tobin-Q augmented APMs. It shows conclusive results for all employed factors including Tobin-Q factor. The table shows number of significant and insignificant portfolio results out of 25 portfolios

Overwhelmingly, the overall findings of TQ-pattern are highly significant and influential in the market as 18/25, 22/25, 21/25 and 22/25 PFs demonstrate highly statistically significant relationship with EPSR using TQ-CAPM, TQ-FF3FM, TQ-C4FM and TQ-FF5FM respectively in the emerging market of Pakistan. The findings of the study not fully endorse the applicability of TQ-FF5FM in PSX as market and profitability patterns show inefficient outputs. The overall average adjusted R-square shows trivial improvement as compare to all models such as 0.0504, 0.15964, 0.1672 and 0.17404 for TQ-adjusted CAPM, FF3FM, C4FM and FF5FM respectively. Consistent with early contributions and recent studies such as Foye (2018) who argued that investment and profitability patterns failed to give justification in Asian equity markets, our findings demonstrate similar results testing TQ-FF5FM in PSX. Similarly, Hanauer and Lauterbach (2019) observed weak results for two-augmented factors of FF5FM using 28 emerging equity markets including PSX.

Table 10 demonstrates the number of significant or insignificant factor results out of total 25 portfolios using time-series OLS regression technique. According to table, market factor determines that CAPM is failed to explain the average excess portfolio returns (AEPR). Similarly, investment factor shows insignificant impact on AEPR as 2 out of 25 portfolios show redundancy in the market. Similar to Lin (2017), the investment factor shows redundancy in the market. The third weaker results represent by momentum factor as 9 out of 25 portfolios exhibit significance results. Conversely, the size pattern represents significant results for approximately 20 out of 25 portfolios. Correspondingly, HML pattern also shows 12 out of 25 significant nexuses with AEPR in the market. More interestingly, the portfolios having high B|M ratio stocks outperform low B|M ratio stocks. Comparative to size pattern, value pattern performance shows weak impact which shows contradictory results as Minović and Živković (2014) referred that as compare to size effect, value effect has more impact on mean stock returns. Moreover, the contributed factor, Tobin-q pattern demonstrates more significant impact as TQ-CAPM, TQ-FF3FM, TQ-C4FM and TQ-FF5FM exhibit 18, 22, 21 and 22 out of 25 portfolios statistically significant relationship with AEPR in PSX.

Model Diagnostics for Model Validation using Gibbons, Ross, and Shanken (1989) Test:

To unveil the suitable and benchmark model for the market, Gibbons, Ross and Shanken (1989) (GRS-1989) test is employed by following Nichol & Dowling (2014); Fama & French (2015, 2016, 2018); Hou, Xue & Zhang (2015); Hou, Mo, Xue, & Zhang, 2021; Fletcher (2019); The performance of the Tobin-q adjusted models in emerging equity market of Pakistan is hypothesized using Gibbons, Ross and Shanken (1989) test.

Table 11: The GRS test findings

Model	Mean alpha	Test stat.	P-value	Mean adj. R2	Mean SE	Mean abs alpha
TQ-CAPM	-0.01148788	2.5837442	0.00008486	0.05032373	0.00320015	0.01148788
TQ-FF3FM	-0.01141321	2.4405731	0.00022404	1.60E-01	0.00302114	0.01141321
TQ-C4FM	-0.01111375	1.9213909	0.00608791	0.16717324	0.00322854	0.01111375
TQ-FF5FM	-0.01106487	2.2601241	0.00073735	0.17404298	0.00303128	0.01106487

Note: Table 11 shows GRS test findings based on average absolute alpha (AAA); average standard error; average adjusted R-square and GRS F-test with p-value.

This study empirically investigates to establish how efficiently Tobin-q augmented standard models such as CAPM, FF3FM, C4FM and FF5FM explain the excess 25 value-weighted portfolio returns in PSX. There are various empirical evaluation techniques followed by many studies regarding model performance to be selected as benchmark model. In this regards, Gibbons, Ross and Shanken (1989) postulated a test based on Wald test which hypothesizes that the absolute alphas produced should be equal or closed to zero in case the model explains the expected excess portfolio returns, Mosoeu, & Kodongo (2020). Correspondingly, if intercept is equal to zero infers model is valid, (Lohano & Kashif, 2018). For overall time period, based on average absolute intercepts (AAA) value, dissimilar with Mosoeu, & Kodongo, 2020; Ekaputra, & Sutrisno, 2020; Kubota & Takehara, 2018; Huyuh, 2017; Chiah et at. 2016, GRS test accepts the TQ-FF5FM for PSX which is comparatively closer to zero. In particular, our findings support TQ-FF5FM as appropriate for pricing stocks returns in emerging market of Pakistan. Moreover, the GRS test findings also support the TQ-C4FM as compare to TQ-FF3FM and TQ-CAPM based on AAA value. The results of TQ-FF5FM reveal that as compare to TQ-FF3FM it outperforms in explaining the portfolio excess returns in emerging market. Similarly, the study of Foye, Mramor, and Pahor (2013) observed failure of the FF3FM in emerging markets.

Conclusion

Empirically, the FF5FM has widely been investigated in developed and emerging equity markets around the globe. The findings of FF5FM, similar to FF3FM, are determining the significant risk premium and providing empirical validation of APM by explaining the cross-section of potential stock returns, around the world. Although, the results evidence mix findings in explaining the average stock returns to justify FF5FM validation. Therefore, the prior studies augmented various additional factors such as momentum and liquidity with FF5FM boost the predictability power of the model. Correspondingly, this study proposes Tobin-q augmented various standard APMs in emerging equity market such as PSX to investigate whether the marginal-factor contributes to measure risk-premium in the domain of asset pricing framework. The study examines the nexus between various factors including market, size (SMB), value (VMG), profitability (RMW), investment (CMA) and particularly Tobin-q (OMU) with excess 25 (value-weighted) portfolio returns in PSX. The prior empirical research studies investigated various asset pricing models with significant augmented anomalies which substantiate FF5FM dominance over FF3FM explaining the variation in CSSR in developed and emerging equity markets around the globe. Although, our results indicate that FF5FM seems relatively effective in PSX except market and investment factors which report statistically insignificant nexus with portfolio excess returns. The results of the study are consistent with previous studies findings using FF5FM in PSX. The contributed TQ factor is priced in the emerging market of Pakistan as overall results comparatively perform better than rest of all market, size, value, momentum, profitability and investment patterns. There are various factors which need to be investigated as additional sixth factor in the PSX to conduct future research in emerging market. The investors and particularly portfolio managers should assume the TQ evaluation pattern while constructing diversified portfolio in emerging equity market of Pakistan.

The future study may include human capital as an additional factor instead of, like (Azam, 2022b). Structural equation modeling is a different approach that might be used in future research to test robustness further such as (Azam, 2022a). Future research may investigate structural breaks before and after Covid-19 utilising the ARCH family techniques as suggested by Azam & Azeem, 2021). The Tobin-q augmented various asset pricing models can be tested such as Fama & French (2018) six-factor model.

Funding

No funding to declare

Conflicts of Interest

The author has no conflict of interest to disclose.

Acknowledgement

None

References

Acaravci, S. K., & Karaomer, Y. (2017). Fama-French five factor model: evidence from Turkey. *International Journal of Economics and Financial Issues*, *7*(6), 130-137.

Acheampong, P., & Swanzy, S. K. (2016). Empirical test of single Factor and multi-factor asset pricing models: Evidence from non-financial firms on the Ghana stock exchange (GSE). *International Journal of Economics and Finance*, *8*(1), 99-110.

Ahlatcıoğlu, A., & Okay, N. (2021). Post-earnings announcement drift: Evidence from Turkey. *Borsa Istanbul Review*.*21*(1), 92-103.

Akhtar, T. (2021). Market multiples and stock returns among emerging and developed financial markets. *Borsa Istanbul Review*, *21*(1), 44-56.

Ali, F. (2022). Testing mispricing-augmented factor models in an emerging market: A quest for parsimony. *Borsa Istanbul Review*, *22*(2), 272-284..

Ali, F., He, R., & Jiang, Y. (2018). Size, value and business cycle variables. The three-factor model and future economic growth: Evidence from an emerging market. *Economies*, *6*(1), 14. 1-24

Ali, F., Khurram, M. U., & Jiang, Y. (2019). The five-factor asset pricing model tests and profitability and investment premiums: Evidence from Pakistan. *Emerging Markets Finance and Trade*, 1-23.

Alroaia, Y. V., Abadi, H. E. M., & Khosravani, A. (2012). The investigation of price-earnings ratio (P/E) and return on stock: The case of Tehran Stock Exchange. *Journal of Asian Development Study*, *1*(2), 35-44.

Ameer, B. & Jamil, Dr. M. (2013). A test of fama and french three factor model in Pakistan equity market. *Global Journal of Management and Business Research*. *13*(7-C), 24-28.

Ardila-Alvarez, D., Forro, Z., & Sornette, D. (2021). The acceleration effect and gamma factor in asset pricing. *Physica A: Statistical Mechanics and its Applications*, *569*, 125367.

Arikan, O., Gozluklu, A. E., Kim, G. H., & Sakaguchi, H. (2019). Primacy in stock market participation: the effect of initial returns on market re-entry decisions. *The European Journal of Finance*, *25*(10), 883-909.

Arsalan, T., & Iqbal, M. (2020). The rol of Risk and Macroeconomic Environment on Stock Returns: Evidence from Banking Sector of Pakistan. *Pakistan Business Review*, *22*(1), 18-29.

Arshad, M. A., Munir, S., Ahmad, B., & Waseem, M. (2019). Do factors matter for predicting high-risk stock returns? Comparison of single-, three-and five-factor CAPM. *International Journal of Financial Engineering*, *6*(02), 1950015.

Azam, M. (2021). Multidimensional Liquidity (Liu, 2006) and Momentum (Carhart, 1997) augmented Fama & French (2015) five-factor model: Evidence from Pakistan. *International Journal of Business and Management Sciences*, *2*(3), 108-130.

Azam, M. (2022a). The CAPM Revisited Using Structural Equation Model: Mediating Role of GDP-Growth in Pakistan Stock Exchange. *Journal of Managerial Sciences*, *16*(1), 93-108.

Azam, M. (2022b). Selectivity and Market-Timing Performance of Mutual Funds Using Human-Capital Augmented Various Asset-Pricing Models: Evidence from Pakistan. *International Journal of Management Research and Emerging Sciences*, *12*(3), 1-31. https://doi.org/10.56536/ijmres.v12i3.173

Azam, M., & Ilyas, J. (2011). An empirical comparison of CAPM and Fama-French Model: A case study of

KSE. *Interdisciplinary Journal of Contemporary Research in Business*, *2*(12), 415-426.

Azam, M., & Azeem, N. (2021). Does COVID-19 Affect Stock Market Volatility?. *International Journal of Business and Management Sciences*, *2*(2), 57-66.

Azimli, A. (2020). Pricing the common stocks in an emerging capital market: Comparison of the factor models. *Borsa Istanbul Review*, *20*(4), 334-346.

Balfoussia, H., & Gibson, H. D. (2019). Firm investment and financial conditions in the euro area: evidence from firm-level data. *Applied Economics Letters*, *26*(2), 104-110.

Bano, S., Chughtai, S., & Awan, T. (2020). Quality minus junk factor: A study on asset pricing dynamics in the equity market of Pakistan. *IBA Business Review*, *15*(2).

Banz, R. W. (1981). The relationship between return and market value of common stocks. *Journal of financial economics*, *9*(1), 3-18.

Basu, S. (1977). Investment performance of common stocks in relation to their price-earnings ratios: A test of the efficient market hypothesis. *The journal of Finance*, *32*(3), 663-682.

Bhandari, L. C. (1988). Debt/equity ratio and expected common stock returns: Empirical evidence. *The journal of finance*, *43*(2), 507-528.

Black, F., Jensen, M. C. & Scholes, M. S. (1972). The capital asset pricing model: Some empirical tests. In: Michael C. Jensen, ed., Studies in the theory of capital markets (Praeger. New York), 79-121.

Bodie, Z., Kane, A., & Marcus, A. (2013). *EBOOK: Essentials of Investments: Global Edition*. McGraw Hill.

Boyle, G. (2005). Risk, expected return, and the cost of equity. New Zealand Economic Papers, 39(2), 181–194.

Callen, J. L. (1988). Estimating the Cost of Equity Capital Using Tobin's q. *The Engineering Economist*, *33*(4), 349-358.

Carhart, M. M. (1997). On persistence in mutual fund performance. *The Journal of finance*, *52*(1), 57-82.

Chhapra, I. U., Rehan, R., Mirza, Z. U. A., & Sohail, A. B. (2020). Understanding Accrual Anomaly: Evidence from Pakistan Equity Market. *KASBIT Business Journal*, *13*(1), 11-24.

Chhapra, I. U., Zehra, I., Kashif, M., & Rehan, R. (2020). Is Bankruptcy Risk a Systematic Risk? Evidence from Pakistan Stock Exchange. *ETIKONOMI*, *19*(1), 51-62.

Chiah, M., Chai, D., Zhong, A., & Li, S. (2016). A Better Model? An empirical investigation of the Fama–French five-factor model in Australia. *International Review of Finance*, *16*(4), 595-638.

Choi, F. D., Hino, H., Min, S. K., Nam, S. O., Ujiie, J., & Stonehill, A. I. (1983). Analyzing foreign financial statements: The use and misuse of international ratio analysis. *Journal of International Business Studies*, *14*(1), 113-131.

Cochrane, J. H. (2011). Presidential address: Discount rates. *The Journal of finance*, *66*(4), 1047-1108.

Davidson, I., Leledakis, G., & Okunev, J. (2002). *Tobin's Q and the Cross Sectional Variation of Stock Returns: Evidence from the London Stock Exchange*. Loughborough: Loughborough University Banking Centre.

Dewandaru, G., Rizvi, S. A. R., Masih, R., Masih, M., & Alhabshi, S. O. (2014). Stock market co-movements: Islamic versus conventional equity indices with multi-timescales analysis. *Economic Systems*, *38*(4), 553-571.

Dirkx, P., & Peter, F. J. (2020). The Fama-French Five-Factor Model Plus Momentum: Evidence for the German Market. *Schmalenbach Business Review*, *72*(4), 661-684.

Dolinar, D., Lončarević, S., & Orlović, Z. (2020, June). Test of the Fama-French five-factor model on the Croatian Stock Market. In *Proceedings of FEB Zagreb International Odyssey Conference on Economics and Business* (Vol. 2, No. 1, pp. 286-297). University of Zagreb, Faculty of Economics and Business.

Douglas, G. W. (1969). Risk in the Equity Markets: An Empirical Appraisal of Market Efficiency. *Yale Economic Essays*, *9*, 3-45.

Ekaputra, I. A., & Sutrisno, B. (2020). Empirical tests of the Fama-French five-factor model in Indonesia and Singapore. *Afro-Asian Journal of Finance and Accounting*, *10*(1), 85-111.

Fama, & French. (1992). The Cross-Section of Expected Stock Returns. *The Journal of Finance*, *47*(2), 427-465.

Fama, & French. (1993). Common risk factors in the returns on stocks and bonds. *Journal of financial Economics*, *33*, 3-56.

Fama, & French. (2015). A five-factor asset pricing model. *Journal of financial Economics*, *116*(1), 1-22.

Fama, E. F., & French, K. R. (2016). Dissecting anomalies with a five-factor model. *The Review of Financial Studies*, *29*(1), 69-103.

Fama, E. F., & French, K. R. (2017). International tests of a five-factor asset pricing model. *Journal of financial Economics*, *123*(3), 441-463.

Fama, E. F., & French, K. R. (2018). Choosing factors. *Journal of financial economics*, *128*(2), 234-252.

Fama, E. F., & MacBeth, J. D. (1973). Risk, return, and equilibrium: Empirical tests. *Journal of political economy*, *81*(3), 607-636.

Feng, J., Wang, Y., & Yin, L. (2017). Oil volatility risk and stock market volatility predictability: Evidence from G7 countries. *Energy Economics*, *68*, 240-254.

Fletcher, J. (2019). Model comparison tests of linear factor models in UK stock returns. *Finance Research Letters*, *28*, 281-291.

Foye, J. (2018). A comprehensive test of the Fama-French five-factor model in emerging markets. *Emerging Markets Review*, *37*, 199-222.

Foye, J., Mramor, D., & Pahor, M. (2013). A respecified fama french three factor model for the new European Union member states. *Journal of International Financial Management & Accounting*, *24*, 3–25.

Frank, M. Z., & Shen, T. (2016). Investment and the weighted average cost of capital. *Journal of Financial Economics*, *119*(2), 300-315.

Fu, L., Singhal, R., & Parkash, M. (2016). Tobin's q ratio and firm performance. *International research journal of applied finance*, *7*(4), 1-10.

Galankashi, M. R., Rafiei, F. M., & Ghezelbash, M. (2020). Portfolio selection: a fuzzy-ANP approach. *Financial Innovation*, *6*(1), 1-34.

Gaunt, C. (2004). Size and book to market effects and the Fama French three factor asset pricing model: evidence from the Australian stockmarket. *Accounting & Finance*, *44*(1), 27-44.

Gibbons, M. R., Ross, S. A., & Shanken, J. (1989). A test of the efficiency of a given portfolio. *Econometrica: Journal of the Econometric Society*, 1121-1152.

Gospodinov, N., Kan, R., & Robotti, C. (2019). Too good to be true? Fallacies in evaluating risk factor models. *Journal of Financial Economics*, *132*(2), 451-471.

Graham, B., Dodd, D. L. F., & Cottle, S. (1934). *Security analysis* (Vol. 452). New York: McGraw-Hill.

Grobys, K., Ruotsalainen, J., & Äijö, J. (2018). Risk-managed industry momentum and momentum crashes. *Quantitative Finance*, *18*(10), 1715-1733.

Guo, B., Zhang, W., Zhang, Y., & Zhang, H. (2017). The five-factor asset pricing model tests for the Chinese stock market. *Pacific-Basin Finance Journal*, *43*, 84-106.

Halliwell, J., Heaney, J. & Sawicki, J. (1999). Size and book to market effects in Australian share markets: a time series analysis. *Accounting Research Journal*, *12*, 122–137.

Hanauer, M. X., & Lauterbach, J. G. (2019). The cross-section of emerging market stock returns. *Emerging Markets Review*, *38*, 265-286.

Hanauer, M. X., & Linhart, M. (2015). Size, value, and momentum in emerging market stock returns: Integrated or segmented pricing?. *Asia-Pacific Journal of Financial Studies*, *44*(2), 175-214.

Hanif, R., Choudhary, S., & Ismail, A. (2017). Implementation of Fama and French Five Factor Model: A Case of Pakistan Stock Exchange. *City University International Journal of Computational Analysis*, *1*(1), 31-38.

Hanif, M. & Bhatti, U., (2010). Validity of capital assets pricing model: Evidence from KSE-Pakistan. *European Journal of Economics, Finance and Administrative Sciences*, (20).

Hansen, L. P. (1982). Large sample properties of generalized method of moments estimators. *Econometrica: Journal of the Econometric Society*, 1029-1054.

Haqqani, K., & Aleem, M. (2020). Testing Liquidity Augmented Fama-French Five-Factor Model in Pakistan Stock Exchange. *Global Economics Review*, *1*, 255-265.

Haqqani, K., & Rahman, W. (2020). The Empirical Test of Fama-French Five-Factor Model: Evidence from Pakistan Stock Exchange. *Global Management Sciences Review*, *5*(3), 32-39.

Haque, A., & Nasir, A. (2018, June). The application of Value at Risk and Expected Shortfall as Controlling Mechanism of Systematic Risk of Pakistani Stock Market. In *Proceedings of Economics and Finance Conferences* (No. 7108551). International Institute of Social and Economic Sciences.

Harvey, C. R., Liu, Y., & Zhu, H. (2016). … and the cross-section of expected returns. *The Review of Financial Studies*, *29*(1), 5-68.

He, W., Shi, X., Chang, Y., & Wu, W. (2021). Pay disparity within top management teams, real earnings management, and firm value: destructive competition or value creation? A moderated mediating model. *Applied Economics Letters*, 1-7.

Hirshleifer, D., & Jiang, D. (2010). A financing-based misvaluation factor and the cross-section of expected returns. *The Review of Financial Studies*, *23*(9), 3401-3436.

Hou, K., Mo, H., Xue, C., & Zhang, L. (2019). Which factors?. *Review of Finance*, *23*(1), 1-35.

Hou, K., Mo, H., Xue, C., & Zhang, L. (2021). An augmented q-factor model with expected growth. *Review of Finance*, *25*(1), 1-41.

Hou, K., Xue, C., & Zhang, L. (2015). Digesting anomalies: An investment approach. *The Review of Financial Studies*, *28*(3), 650-705.

Huang, T. L. (2019). Is the Fama and French five-factor model robust in the Chinese stock market?. *Asia Pacific Management Review*, *24*(3), 278-289.

Huberman, G., & Kandel, S. (1987). Mean-variance spanning. *The Journal of Finance*, *42*(4), 873-888.

Huynh, T. D. (2017). Conditional asset pricing in international equity markets. *International Review of Economics & Finance*, *49*, 168-189.

Ibrahim, A., Awan, T., & Khan, M. Y. (2018). Empirical investigation of socially responsible investments in Pakistani firms using Carhart Four Factor model. *IBA Business Review*, *13*(2).

Ishtiaq, M. (2019). Application of Fama-French Five Factor Model in Stock Pricing: Evidence from

Emerging Market. *Pacific Business Review International*, *11*(7).

Ishaq, M., Islam, Y., & Ghouse, G. (2021). Tobin's Q as an Indicator of Firm Performance: Empirical Evidence from Manufacturing Sector Firms of Pakistan. *International Journal of Economics & Business Administration (1)*, 425-441.

Jaffe, J., Keim, D. B., & Westerfield, R. (1989). Earnings yields, market values, and stock returns. *The Journal of Finance*, *44*(1), 135-148.

Jan, M. N., & Ayub, U. (2019). Do the Fama and French Five-Factor model forecast well using ANN?. *Journal of Business Economics and Management*, *20*(1), 168-191.

Jan, S. U., Iqbal, S., & Aamir, A. (2021). Comparing CAPM and FAMA French for Predicting Stock Returns: New Evidence from Pakistan Stock Exchange. *Ilkogretim Online*, *20*(3).

Jareño, F., O González, M. D. L., Tolentino, M., & Rodríguez, S. (2018). Interest rate sensitivity of Spanish companies. An extension of the Fama-French five-factor model. *Acta Oeconomica*, *68*(4), 617-638.

Javed, A. (2020). Dividend Price Ratio and Stock Return: Evidence from Pakistan, India and China. *International Journal of Psychosocial Rehabilitation*, *24*(03).

Javid, A. Y., & Ahmad, E. (2008). Testing multifactor capital asset pricing model in case of Pakistani market.114-138.

Kaldor, N. (1966). Marginal productivity and the macro-economic theories of distribution: Comment on Samuelson and Modigliani. *The Review of Economic Studies*, *33*(4), 309-319.

Kassimatis, K. (2008). Size, book to market and momentum effects in the Australian stock market. *Australian Journal of Management*, *33*(1), 145-168.

Kassimatis, K. (2011). Risk aversion with local risk seeking and stock returns: evidence from the UK market. *Journal of Business Finance & Accounting*, *38*(5-6), 713-739.

Kaya, E. (2020). Relative performances of asset pricing models for BIST 100 index. *Spanish Journal of Finance and Accounting/Revista Española de Financiación y Contabilidad*, 1-22.

Khan, U. E., & Iqbal, J. (2021). The Relationship between Default Risk and Asset Pricing: Empirical Evidence from Pakistan. *The Journal of Asian Finance, Economics and Business*, *8*(3), 717-729.

Kubota, K., & Takehara, H. (2018). Does the Fama and French five-factor model work well in Japan?. *International Review of Finance*, *18*(1), 137-146.

Lau, S. T., Lee, C. T., & McInish, T. H. (2002). Stock returns and beta, firms size, E/P, CF/P, book-to-market, and sales growth: evidence from Singapore and Malaysia. *Journal of multinational financial management*, *12*(3), 207-222.

Leal, C. C., Armada, M. J. R., & Loureiro, G. (2018). Individual investors repurchasing behaviour: evidence from the Portuguese stock market. *The European Journal of Finance*, *24*(11), 976-999.

Lee, H. S., Cheng, F. F., & Chong, S. C. (2016). Markowitz portfolio theory and capital asset pricing model for Kuala Lumpur stock exchange: A case revisited. *International Journal of Economics and Financial Issues*, *6*(3S), 59-65.

Li, Q., Vassalou, M., & Xing, Y. (2006). Sector investment growth rates and the cross section of equity returns. *The Journal of Business*, *79*(3), 1637-1665.

Li, X., Wu, X., & Zhou, W. (2017). Optimal stopping investment in a logarithmic utility-based portfolio selection problem. *Financial Innovation*, *3*(1), 1-10.

Liammukda, A., Khamkong, M., Saenchan, L., & Hongsakulvasu, N. (2020). The Time-Varying Coefficient Fama-French Five Factor Model: A Case Study in the Return of Japan Portfolios. *The Journal of Asian Finance, Economics, and Business*, *7*(10), 513-521.

Lin, Q. (2017). Noisy prices and the Fama–French five-factor asset pricing model in China. *Emerging Markets Review*, *31*, 141-163.

Lintner. (1965). Security prices, risk, and maximal gains from diversification. *The Journal of Finance*, *20*(4), 587-615.

Liu, J., Stambaugh, R. F., & Yuan, Y. (2019). Size and value in China. *Journal of Financial Economics*, *134*(1), 48-69.

Liu, S. (2020, December). Analysis of COVID-19 on Service Industry Based on Fama and French Five-Factor Model. In *2020 Management Science Informatization and Economic Innovation Development Conference (MSIEID)* (pp. 154-157). IEEE.

Liu, W. (2006). A liquidity-augmented capital asset pricing model. *Journal of financial Economics*, *82*(3), 631-671.

Lohano, K., & Kashif, M. (2018). Testing asset pricing models on the Pakistan Stock Exchange. *IBA Business Review*, *13*(2), 1-19.

Madsen, J. (2003). Equity prices, share price valuation, crashes and depressions. *Nationalokonomisk Tidsskrift*, *141*, 3-34.

Markowitz, H. (1952). Portfolio Selection. *The Journal of Finance*, *7*(1), pp. 77-91.

Mengoli, S. (2004). On the source of contrarian and momentum strategies in the Italian equity market. *International Review of Financial Analysis*, *13*(3), 301-331.

Merton, R. C. (1973). An intertemporal capital asset pricing model. *Econometrica: Journal of the Econometric Society*, 867-887.

Merton, R. C. (1987). A simple model of capital market equilibrium with incomplete information.

Miller, M. H., & Scholes, M. (1972). Rates of return in relation to risk: A reexamination of some recent findings. *Studies in the theory of capital markets*, *23*, 47-48.

Minović, J., & Živković, B. (2014). CAPM augmented with liquidity and size premium in the Croatian stock market. *Economic research-Ekonomska istraživanja, 27*(1), 191-206.

Mirza, N., & Reddy, K. (2017). Asset pricing in a developing economy: Evidence from Pakistan. *Economics Bulletin*, *37*(4), 2483-2495.

Mirza, N., & Shahid, S. (2008). Size and Value Premium in Karachi Stock Exchange. *Lahore Journal of Economics*, *13*(2), 1-26.

Mosoeu, S., & Kodongo, O. (2020). The Fama-French five-factor model and emerging market equity returns. *The Quarterly Review of Economics and Finance*. DOI: https://doi.org/10.1016/j.qref.2020.10.023

Mossin, J. (1966). Equilibrium in a capital asset market. *Econometrica: Journal of the Econometric Society*, 768-783.

Mukherji, S., Dhatt, M. S., & Kim, Y. H. (1997). A fundamental analysis of Korean stock returns. *Financial Analysts Journal*, *53*(3), 75-80.

Munir, S., Sajjad, M., Humayon, A., & Chani, M. (2020). Determination of stock returns using five-factor CAPM: evidence from PSX. *International Transaction Journal of Engineering, Management, & Applied Sciences & Technologies*, *11*(8), 1-10.

Nichol, E., & Dowling, M. (2014). Profitability and investment factors for UK asset pricing models. *Economics Letters*, *125*(3), 364-366.

Novy-Marx, R. (2013). The other side of value: The gross profitability premium. *Journal of financial economics*, *108*(1), 1-28.

Pástor, Ľ., & Stambaugh, R. F. (2003). Liquidity risk and expected stock returns. *Journal of Political economy*, *111*(3), 642-685.

Peng, L., & Xiong, W. (2006). Investor attention, overconfidence and category learning. *Journal of Financial Economics*, *80*(3), 563-602.

Penman, S. H. (1992). Return to fundamentals. *Journal of Accounting, Auditing & Finance*, *7*(4), 465-483.

Pietrovito, F. (2016). Do price-earnings ratios explain investment decisions better than Tobin'sq? Evidence from German firm-level data. *Applied Economics, 48*(34), 3264-3276.

Pojanavatee, S. (2020). Tests of a Four-Factor Asset Pricing Model: The Stock Exchange of Thailand. *The Journal of Asian Finance, Economics, and Business*, *7*(9), 117-123.

Qin, S., Luo, W., & Wang, Y. (2021). Policy uncertainty and firm-level investment: Evidence from public-private partnership market in China. *Applied Economics Letters*, 1-7.

Ragab, N. S., Abdou, R. K., & Sakr, A. M. (2019). A Comparative Study between the Fama and French Three-Factor Model and the Fama and French Five-Factor Model: Evidence from the Egyptian Stock Market. *International Journal of Economics and Finance*, *12*(1), 52-69.

Rendón, J. A. (2020). Foreign exchange risk in stock returns. *International Journal of Finance & Economics*, *25*(3), 430-443.

Racicot, F. E., & Rentz, W. F. (2016). Testing Fama–French's new five-factor asset pricing model: evidence from robust instruments. *Applied Economics Letters*, *23*(6), 444-448.

Racicot, F. E., & Rentz, W. F. (2017). A panel data robust instrumental variable approach: A test of the new Fama-French five-factor model. *Applied Economics Letters*, *24*(6), 410-416.

Racicot, F. É., Rentz, W. F., & Théoret, R. (2018). Testing the new Fama and French factors with illiquidity: A panel data investigation. *Finance, 39*(3), 45-102.

Rizwan, S., Shaikh, S. J., & Shehzadi, M. (2013). Validity of capital assets pricing model (CAPM): evidence from cement sector of Pakistan listed under Karachi stock exchange. *Kuwait Chapter of the Arabian Journal of Business and Management Review*, *2*(6), 66.

Rosenberg, B., Reid, K., & Lanstein, R. (1985). Persuasive evidence of market inefficiency. *The Journal of Portfolio Management*, *11*(3), 9-16.

Ross, S. (1976). The Arbitrage Theory of Capital Asset Pricing. *Journal of Economic Theory*, *13*, 341-360.

Roy, R., & Shijin, S. (2018). A six-factor asset pricing model. *Borsa Istanbul Review*, *18*(3), 205-217.

Sadhwani, R. Bhayo M. R. & Bhutto, N. A. (2019). A Test of the Five-Factor Model in Pakistan. *Pacific Business Review International*,*11*(7), 42-52.

Sehrawat, N., Kumar, A., Nigam, N. K., Singh, K., & Goyal, K. (2020). Test of capital market integration using Fama-French three-factor model: Empirical evidence from India. *Investment Management & Financial Innovations*, *17*(2), 113-127.

Shah, S. H. A., Shah, A., Khan, M. K., & Ullah, H. (2021). The Risk and Return Relations: New Evidence from Pakistani Stock Market. *Journal of Accounting and Finance in Emerging Economies*, *7*(1), 195-204.

Shaharuddin, S. S., Lau, W. Y., & Ahmad, R. (2018). Is the Fama french three-factor model relevant? Evidence from Islamic unit trust funds. *The Journal of Asian Finance, Economics, and Business*, *5*(4), 21-34.

Shaikh, S. A., Ismail, M. A., Ismail, A. G., Shahimi, S., & Shafiai, M. H. M. (2019). Cross section of stock returns on Shari'ah-compliant stocks: evidence from Pakistan. *International Journal of Islamic and Middle Eastern Finance and Management*.

Sharpe. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *The Journal of Finance*, *19*(3), 425-442.

Shoaib, A., Siddiqui, M. A. & Ayub, M. (2020). Revisiting Fama-French Model through Alternative Size Measures: Evidence from South Asian Countries. *Abasyn University Journal of Social Sciences*, *13*(1), 137-151.

Situm, M. (2021). Determination of expected cost of equity with the CAPM: Theoretical extension using the law of error propagation. *Managerial and Decision Economics*, *42*(1), 77-84.

Škrinjarić, T. (2020). Dynamic portfolio optimization based on grey relational analysis approach. *Expert systems with applications*, *147*, 113207.

Stattman, D. (1980). Book Values and Stock Returns, The Chicago MBA: *A Journal of Selected Papers*. 4, 25-45

Stowe, J. D., Robinson, T. R., Pinto, J. E., & McLeavey, D. W. (2007). *Equity asset valuation* (Vol. 4). John Wiley & Sons.

Tharwani, R., Chhapra, I. U., Shakeel, S., & Sarwat, S. (2020). The Gross Profitability Premium: An Empirical Examination in Pakistan Stock Exchange (PSX). *Global Management Journal for Academic & Corporate Studies (GMJACS)*, 65.

Titman, S., Wei, K. J., & Xie, F. (2004). Capital investments and stock returns. *Journal of financial and Quantitative Analysis*, *39*(4), 677-700.

Tobin, J. (1970). On limiting the domain of inequality. *The Journal of Law and Economics*, *13*(2), 263-277.

Treynor, J. L. (1962), Toward a theory of market value of risky assets, the first paper on CAPM, yet rarely cited and often incorrectly referred to as 'Treynor (1961)'

Tsai, J. F., Mai, N. T., & Bui, D. G. (2021). Managerial ability, financial constraints, and the value of cash holding. *Applied Economics Letters*, 1-7.

Vassalou, M. (2003). News related to future GDP growth as a risk factor in equity returns. *Journal of financial economics*, *68*(1), 47-73.

Wang, L., & Xiong, X. (2021). The impact of herding behaviour of Chinese securities investment funds on firm value: the mediating effects of stock price informativeness and agency cost. *Applied Economics Letters*, 1-8.

Watanabe, A., Xu, Y., Yao, T., & Yu, T. (2013). The asset growth effect: Insights from international equity markets. *Journal of Financial Economics*, *108*(2), 529-563.

Wernerfelt, B., & Montgomery, C. A. (1988). Tobin's q and the importance of focus in firm performance. *The American Economic Review*, 246-250.

Wolfe, J., & Sauaia, A. C. A. (2003). The Tobin q as a company performance indicator. In *Developments in Business Simulation and Experiential Learning: Proceedings of the Annual ABSEL conference* (Vol. 30).

Wu, W., Chen, J., Xu, L., He, Q., & Tindall, M. L. (2019). A statistical learning approach for stock selection in the Chinese stock market. *Financial Innovation*, *5*(1), 1-18.

Wu, M., Imran, M., Feng, Y., Zhang, L., & Abbas, M. (2017). Review and validity of capital asset pricing model: evidence from Pakistan stock exchange. *International Research in Economics and Finance*, *1*(1), 21.

Xie, S., & Qu, Q. (2016). The three-factor model and size and value premiums in China's stock market. *Emerging Markets Finance and Trade*, *52*(5), 1092-1105.

Zada, H., Hassan, A., Khan, M. Z., & Afeef, M. (2019). An Appropriate Investment Portfolio for Stock Market Investors: Evidence from Pakistan. *Sarhad Journal of Management Sciences*, *5*(2), 343-356.

Zada, H., Rehman, M. U., & Khwaja, M. G. (2018). Application of Fama And french five factor model of asset pricing: Evidence from Pakistan stock market. *International Journal of Economics, Management and Accounting*, *26*(1), 1-23.

Zaremba, A., & Czapkiewicz, A. (2017). Digesting anomalies in emerging European markets: A comparison of factor pricing models. *Emerging Markets Review*, *31*, 1-15.

Zhang, L., Li, Y., Huang, Z., & Chen, X. (2018). Stock liquidity and firm value: evidence from China. *Applied Economics Letters*, *25*(1), 47-50.

Zhang, Q., Saqib, Z. A., Saqib, K. A., Mahmood, S., & Cao, M. (2019, July). Alternative Three Factor Model for Asset Pricing for the Investment, Market and Profitability Premium in Emerging Markets: An Evidence from Pakistan's Stock Exchange. In *2019 16th International Conference on Service Systems and Service Management (ICSSSM)* (pp. 1-6). IEEE

Zheng, D., Chiang, T. C., & Nelling, E. (2020). Tests of Multifactor Asset Pricing Models in Asian Stock Markets. In *Emerging Market Finance: New Challenges and Opportunities*. Emerald Publishing Limited.